Intangible Capital and Stock Prices

Nan Li[*†]

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Abstract

This paper develops a two-sector dynamic stochastic general equilibrium model to measure intangible capital stock and studies the implied riskiness of market value of capital. The equilibrium of the economy is characterized by a state-space representation of dynamic system. Kalman filter algorithm is used to produce an estimate of the value of intangible capital stock based on the observed data on macroeconomic variables and asset prices. With modest capital adjustment cost, the model implies that significant amount of intangible capital is accumulated during past 50 years in US economy but the growth of intangible capital in the last decade is not as fast as the estimates of Hall (2001). Variation in intangible capital estimated from aggregate macroeconomic variables, accounts for almost half of the variability in the market-to-book ratio of nonfinancial and nonfarm corporate firms.

Keywords: Intangible capital; Investment-specific technological progress; Tobin's q; Kalman Filter;

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[†] Department of Finance and Accounting, NUS Business School, National University of Singapore, Singapore 117596 Tel.: 65-6516-1403 *E-mail address*: biznl@nus.edu.sg

1. Introduction

Intangible assets are nonphysical productive assets that create corporate value and economic growth. Some argue that the value of intangible capital became an increasingly important component of the corporate value as reflected on the stock market. Despite its increasing importance in the economy, intangible capital is routinely expensed in the financial reports and remains absent from corporate balance sheets and national income and product accounts (NIPA). However, it is important to value this intangible or unmeasured part of the capital, which includes capital stock results from investment in research and development, firm-specific human capital, firm-specific organizational capital, patents, brand name and etc.

As argued in Hansen, Heaton, and Li (2005), intangible capital can be inferred by computing the residual contribution to production after all other measured inputs are accounted for. The quantity of intangible capital can be inferred from information on marginal product of capital which can be deducted from asset returns. The value of intangible capital can be estimated by comparing the asset market value measured by market value of firm, to the value of physical capital. However, in an economic model with physical capital as the only type of capital, intangible capital is buried in the Solow residual along with measurement error, omitted information and model approximation error. This paper develops a two-sector dynamic stochastic general equilibrium model in which intangible capital is model explicitly as an input in production, the value and magnitude of intangible stock is estimated from the observed data on macroeconomic variables and asset prices.

Since time series of both macroeconomic variables and asset prices are used to make inferences about intangible capital, it is important to develop a model that is consistent with the salient facts in both macroeconomics and asset markets. Greenwood, Hercowitz, and Krusell (1997) find that the investment specific technological progress is an important source of the business cycle shocks and long-run growth. Christiano and Fisher (1998) show that a two-sector model, in which technological progress in the production of investment goods exceeds that in the rest of the economy, is consistent with the procyclicality of stock prices and the countercyclicality of investment good prices. Whelan (2003) argues that this type of two-sector model do much better than the existing one-sector models on explaining crucial long-run properties of US macroeconomic data. Following the line of this literature, we develop a two-sector model with investment specific technological shock and exploit the information on investment good prices and stock prices to measure intangible capital.

However, the standard stochastic growth model is known to produce too little variability in physical returns relative to security market counterparts. In the one-sector setting, the relative price of investment goods becomes unity and the only source of variability is the marginal product of capital. Inducing variability in this term by variability in the technology shock process generates aggregate quantities such as output and consumption that are too variable. The supply of capital is less elastic when adjustment costs exist, hence models with adjustment costs can deliver larger return variability than the standard stochastic growth model. This motivated Cochrane (1991) and Jermann (1998) to include adjustment costs to physical capital in their attempts to generate interesting asset market implications in models of aggregate fluctuations. As an alternative, Boldrin, Christiano, and Fisher (2001) study a two-sector model with limited mobility of capital across technologies. In our environment, limited mobility between physical and intangible capital could be an alternative source of aggregate return variability.

Several other papers estimate the value of intangible capital, focusing on the relation between intangible capital and stock market. Atkeson and Kehoe (2002) and others exploit the homogeneity of the production function and Euler's Theorem to write output as sum of products of inputs and its marginal productivity. Thus the contribution of intangible capital to output is measured by "residual'. This approach avoids the need to directly measure rental income to measured capital, but it instead requires measures of the physical returns, physical depreciation scaled by value appreciation and the relative value of tangible capital to income.

However, the physical return to measured capital is not directly observed. Atkeson and Kehoe (2002) take a steady-state approximation, implying that returns should be equated to measure the importance of intangible capital in manufacturing. Income shares and price appreciation are measured using time series averages. Given the observed heterogeneity in average returns, there is considerable ambiguity as to which average return to use.

McGrattan and Prescott (2000) use a similar method along with steady-state calculation and a model in which investment price is always unit of infer the intangible capital stock. Instead of using security market returns or historical return average of these returns, they construct physical returns, presuming that the noncorporate sector does not use intangible capital in production. Rather than making this seemingly hard-to-defend restriction, we explicitly model the link between physical return and security market return directly in the two-sector model and estimate the time series of intangible capital based on the observed variables. However, the practical question of which security market return to use would be still be present. The measurement problem is made simpler by the fact that it is the composite returns that needs to computed and not the individual return on measured capital. The implied one-period return to equity and bond holder is combined as in Hall (2001), but computing the appropriate one-period return for bond holders may still be problematic.

In contrast to Atkeson and Kehoe (2002) and McGrattan and Prescott (2000), uncertainty is central in the analysis of Hall (2001), who models the tangible and intangible capital stock are perfect substitutes. Instead of returns, Hall (2001) uses asset values to deduce time series for the aggregate capital stock and corresponding shadow valuation of that stock. However, evidence from empirical finance on return heterogeneity indicates important differences between returns to tangible and intangible components of the capital stocks. This suggests the consideration of models in which intangible capital differs from tangible capital in ways that might have important consequences for measurement. In this paper, the tangible and intangible capital stock are allowed to contribute different share to the production and the return heterogeneity is explicitly modelled.

This paper is organized as follows. Section 2 presents the two-sector stochastic growth model with intangible capital, in which technological progress in the production of investment goods exceeds that in the production of consumption goods. Section 3 outlines the method to solve and estimate the model using Kalman filtering algorithm. The value of intangible capital is also estimated for the models of Hall (2001) comparison. Section 4 discusses the model's implications for asset prices and returns. Section 5 concludes.

2. The Two Sector Model

In the literature on investment specific technology progress, it is well documented that there is a downward trend in relative price of investment goods while real investment has grown faster than real consumption in postwar US economy, see for example Greenwood, Hercowitz, and Krusell (1997) and Whelan (2003). As shown in Figure 1[1], the ratio of nominal private fixed investment to nominal consumption of consumer nondurables and services has been stable, while the relative price of investment to price of consumption has declined since late 1950s and especially after 1980, and the ratio of real investment to real consumption has trended upwards since late 1950s and has risen dramatically since 1991. The average growth rate of real private fixed nonresidential investment is 4.2% per year over the period 1947-2009, 0.8% per year faster than real consumption. If we were to exploit the information on relative prices of investment goods to measure intangible capital, we need a two-sector model that allows investment goods to grow faster than other outputs. On the other hand, if we were to exploit the information on stock prices at the same time, we need a model that produces different cyclical behavior of stock price and investment good prices. Hence, adjustment cost is needed to drive a wedge between the marginal cost of installed capital, which is measured by stock prices, and the marginal cost of new capital measured by investment good prices.

The model developed in this section resembles that of Greenwood, Hercowitz, and Krusell (1997), Christiano and Fisher (1998) and Whelan (2003) in that it focuses on a two-sector economy with one sector growing faster than the other. The most important difference is that intangible capital is added as a distinct productive factor in the production of both sectors, and the shares of the two types of capital may differ different sectors. The two-sector model presented here also shares some common features with that of **?**) in that the share of intangible capital is different across sectors, but **?**) restrict that the noncorporate sector does not use intangible capital in production, and the relative price of investment is always 1.

2.1. Preferences and Technology

The economy is populated with the infinitely lived households. Household preferences are given as following

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[C_t(1-N_t)^\theta]^{1-\gamma} - 1}{1-\gamma}, \gamma \geq 0, 0 < \theta < 1, 0 < \beta < 1 \tag{1}$$

where C_t denotes consumption at date t, N_t denotes the sum of the factions of productive time allocated to the two production sectors, whereas $1 - N_t$ is the fraction of productive time allocated to leisure. The parameter γ is the coefficient of the relative risk aversion for intertemporal wealth gambles, where wealth is measured in terms of the composite commodity $C(1-N)^\theta$. The parameter θ is the atemporal elasticity of the substitution between consumption and leisure. The larger is θ, the more the household is willing to substitute consumption for leisure. β is the discount factor, and it measures the impatience to consume.

In this model, the consumption and investment goods are produced in separate sectors. Sector 1 produces consumption goods with constant return to scale technology as following,

$$C_t \leq X_{1,t} K_{1m,t}^{\alpha_{1m}} K_{1u,t}^{\alpha_{1u}} {N_{1,t}}^{1-\alpha_{1m}-\alpha_{1u}} \tag{2}$$

where $K_{1m,t}$ and $K_{1u,t}$ are the physical capital and intangible capital carried into date t and used in the consumption production sector, and $N_{1,t}$ is the labor input in this sector. $X_{1,t}$

is the aggregate productivity shock, and the logarithm of $X_{1,t}$ follows a random walk with positive drift, that is,

$$\log X_{1,t} = g_1 + \log X_{1,t-1} + \varepsilon_{1,t} \tag{3}$$

New investment goods in physical capital and intangible capital are produced in sector 2. The technology for producing new investment goods at date t is

$$I_{m,t} + I_{u,t} \leq X_{1,t} K_{2m,t}^{\alpha_{2m}} K_{2u,t}^{\alpha_{2u}} (X_{2,t} N_{2,t})^{1-\alpha_{2m}-\alpha_{2u}} \tag{4}$$

where $I_{m,t}$ and $I_{u,t}$ are the investment goods in physical and intangible capital produced at date t, respectively. $K_{2m,t}$ and $K_{2u,t}$ are the capital inputs, and N_{2t} is the labor input in sector 2 at date t. $X_{2,t}$ is the productivity shock specific to the investment production sector. The logarithm of $X_{2,t}$ also follows a random walk with positive drift

$$\log X_{2,t} = g_2 + \log X_{2,t-1} + \varepsilon_{2,t} \tag{5}$$

Here $\{\varepsilon_{1,t}\}$ and $\{\varepsilon_{2,t}\}$ are sequences of *i.i.d.* zero-mean normally distributed random variables, which are independent of each other and over time, and have standard deviations of σ_1 and σ_2, respectively. g_1 and g_2 are the mean growth rates of technological changes $X_{1,t}$ and $X_{2,t}$, respectively.

To keep things as simple as possible without loss of the interesting aspect of the model, we assume that the total capital share is same in two sectors, that is

$$\alpha_{1m} + \alpha_{1u} = \alpha_{2m} + \alpha_{2u} = \alpha$$

where $0 < \alpha < 1$, is the total capital share.

There are adjustment costs associated with the installation of new investment goods of both types of capital, so capital stock evolves according to

$$K_{x,t+1} = (1 - \delta_x) K_{x,t} + \phi_x \left(\frac{I_{x,t}}{K_{x,t}}\right) K_{x,t} \tag{6}$$

where δ_x is the depreciation rate and $K_{x,t}$ is the total capital stock of type x, for $x = m, u$, that is,

$$K_{x,t} = K_{1x,t} + K_{2x,t} \tag{7}$$

$\phi_x(\cdot)$ is a positive concave function in investment-capital ratio specified as follows

$$\phi_x(\frac{I_{x,t}}{K_{x,t}}) = \frac{a_{x,1}}{1 - 1/\xi_x} \left(\frac{I_{x,t}}{K_{x,t}}\right)^{1-1/\xi_x} + a_{x,2}$$

for $\xi_x \geq 0$. $a_{1,x}$ and $a_{2,x}$ are chosen so that the steady-state growth path is invariant to the specification of ${\xi_x}^2$. The adjustment cost functions are positive concave in investment-capital ratio. Following Grunfeld (1960), Lucas and Prescott (1971) and Hayashi (1982), capital adjustment costs have provided a framework to study the relation between the value of firm and its capital stock. The specification of adjustment costs associated with installation of capital stock allows the shadow price of installed capital to be different from the price of a unit of new investment goods, that is, Tobin's q may vary over time. The parameter ξ_x is the elasticity of investment, $I_{x,t}$ with respect to Tobin's q associated with the capital of type x, for $x = m, u$. The smaller is ξ_x, the more costly it is to change the capital stock. It is easy to verify that when $\xi_x = \infty$, this formulation reduce to the conventional linear capital accumulation equation without adjustment cost.

2.2. Steady-State Growth Rates

In this economy, along the steady-state growth path, investment and capital stock of both types follow the same stochastic trend, which is different from the stochastic trend of consumption. Denote the growth rate of investment and capital stock as g_k and that of consumption as g_c, taking log-differences of equations (2) and (4), we get

$$\begin{aligned} g_c &= g_1 + \alpha_{1m} g_k + \alpha_{1u} g_k \\ g_k &= g_1 + \alpha_{2m} g_k + \alpha_{2u} g_k + (1-\alpha) g_2 \end{aligned}$$

The solutions to this set of equations are

$$
\begin{aligned}
g_c &= \frac{1}{1-\alpha} g_1 + \alpha g_2 \\
g_k &= \frac{1}{1-\alpha} g_1 + g_2
\end{aligned}
$$

so the stochastic trend followed by investment and capital stock is $X_{1t}^{1/(1-\alpha)} X_{2t}$, and the stochastic trend of consumption is $X_{1t}^{1/(1-\alpha)} X_{2t}^{\alpha}$. The growth rate of real investment goods exceeds that of real consumption by $(1-\alpha)g_2$, and the relative price of the investment goods has a downward stochastic trend of $X_{2t}^{-(1-\alpha)}$.

2.3. Transformed Economy and Log-linearization

For this two-sector model, the second welfare theorem holds, so the quantities in a competitive equilibrium of the models can be computed by solving the social planner's problem, and the relative prices can be computed using the Lagrangian multipliers from a solution to the planner's problem.

To solve the social planner's problem, first we need to transform the nonstationary economy into a stationary system. The transformation involves dividing all variables by their stochastic trends except for labor inputs, such as$c_t = \dfrac{C_t}{X_{1t}^{1/(1-\alpha)} X_{2t}^{\alpha}}$, $k_{x,t} = \dfrac{K_{x,t}}{X_{1t}^{1/(1-\alpha)} X_{2t}}$, $i_{x,t} = \dfrac{I_{x,t}}{X_{1t}^{1/(1-\alpha)} X_{2t}}$ (for $x = m, u$) and etc., where variables in lower case denote the transformed variables. In this transformed economy, the social planner maximizes (1) subject to (2)- (7) with upper-case variables replaced by their lower-case counterparts, but with two exceptions. First, the capital accumulation equation (6) becomes

$$
\exp(g_k + \varepsilon_{k,t+1}) k_{x,t+1} = (1-\delta_x) k_{x,t} + \phi_x\left(\frac{i_{x,t}}{k_{x,t}}\right) k_{x,t}, \text{ for } x = m, u \tag{8}
$$

where $\varepsilon_{k,t} = \frac{1}{1-\alpha}\varepsilon_{1,t} + \varepsilon_{2,t}$. Second, the discount factor β is altered as the result of transformation of consumption in the preference specification. The discount factor in the transformed economy is

$$\beta^* = \beta \exp((1-\gamma)g_c)$$

The solution to the social planner's problem is characterized by the first-order conditions, which are nonlinear functions of the transformed variables. Following King, Plosser, and Rebelo (1988) and King, Plosser, and Rebelo (2002), we solve this nonlinear problem by first loglinearizing first-order conditions around steady state and then solving a first-difference linear dynamic system.

Denote the percentage deviation from steady-state growth path of the variables in the transformed economy using a circumflex, that is, $\hat{c}_t = \log(c_t/c)$, $\hat{k}_{x,t} = \log(k_{x,t}/k_x)$, $\hat{\imath}_{x,t} = \log(i_{x,t}/i_x)$ (for $x = m, u$) and etc., where c, k_x, i_x denote the steady-state value of consumption, capital stock and investment in the transformed economy. Then the approximate dynamics of the model economy can be represented by a first-order autoregressive system in $\hat{k}_t = \begin{bmatrix} \hat{k}_{mt} & \hat{k}_{ut} \end{bmatrix}$ as follows,

$$\hat{k}_t = M_1 \hat{k}_{t-1} + M_2 \varepsilon_t \tag{9}$$

where $\varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} & \varepsilon_{2t} \end{bmatrix}$ is the vector of productivity shocks. The square matrix M_1 governs the dynamic of the system. The eigenvalues of M_1 have modulus less than one, that is, the system of state variable $\hat{k}_t$ is stationary.

Deviation from steady state path of all the other transformed hence stationary macroeconomic variables, such as consumption, investment, prices, labor income are linear functions of the state variables $\hat{k}_{t,}$

$$\hat{c}_t = \pi_{ck}\hat{k}_t,\ \hat{\imath}_{xt} = \pi_{i_xk}\hat{k}_t,\ \hat{p}_{i,t} = \pi_{pk}\hat{k}_t,\ w\hat{n}_t = \pi_{wnk}\hat{k}_t, \tag{10}$$

where the coefficients in π, M_1 and M_2 are functions of parameters of preferences and technology.

Given the dynamics of the transformed (stationary) economy, it is straightforward to get the dynamics of the original economy. The variables in the original economy can be specified as

$$\log K_{x,t} = \log X_{k,t} + \log(k_x) + \hat{k}_{x,t}, \text{ for } x = m, u$$
$$\log C_t = \log X_{c,t} + \log(c) + \pi_{ck}\hat{k}_t$$
$$\log I_{x,t} = \log X_{k,t} + \log(i_x) + \pi_{i_x k}\hat{k}_t, \text{ for } x = m, u$$
$$\log P_{i,t} = \log X_{p,t} + \log(p) + \pi_{pk}\hat{k}_t$$
$$\log(WN_t) = \log X_{c,t} + \log(wn) + \pi_{wnk}\hat{k}_t$$

where $\log X_{c,t} = \frac{1}{1-\alpha}\log X_{1,t} + \alpha \log X_{2,t}$, $\log X_{k,t} = \frac{1}{1-\alpha}\log X_{1,t} + \log X_{2,t}$ and $\log X_{p,t} = -(1-\alpha)\log X_{2,t}$ are the stochastic trends for consumption, capital stock (and investment) and investment goods price, respectively.

Then we can represent the model solution in a state-space system as following

$$\text{state equation: } s_{t+1} = Fs_t + Gu_t + \Gamma\varepsilon_{t+1} \tag{11}$$
$$\text{observation equation: } y_t = A'u_t + H's_t + w_t$$

where s_t is the vector of state variables and y_t is the vector of observed variables. In the economy considered in this paper, s_t contains the logs of capital stock and productivity shock, that is

$$s_t = \begin{bmatrix} \log K_t & \log X_t \end{bmatrix}'$$

y_t contains the logs of observable variables such as consumption, investment in physical capital, relative price of investment goods, labor income and market value of total capital. u_t is constant, w_t is the vector of measurement error. F, G, Γ, A, H are matrix with their elements as functions of π, M_1, M_2 and etc[3].

We use DYNARE package solve the model using log-linearization outlined above and estimate the parameters using Maximum Likelihood Estimation method. We obtain the state-space system (11) from the model solution provided by DYNARE. The time series of intangible capital stock and other unobserved variables are imputed through Kalman filter algorithm. However, the moments of asset returns are simulated in the system where the model is solved using second-order perturbation methods, due to the certainty equivalence problem of the log-linearization.

2.4. Prices and Rates of Return

Before discussing the estimation and simulation of the model, we would like to summarize property of the prices and rates of return in this economy of our interests.

2.4.1. Prices of investment goods and capital

Price of new investment goods at date t, $P_{i,t}$, equals the ratio of the shadow price of the investment goods to the shadow price of the consumption goods, where the shadow prices are given as the Lagrangian multipliers from the planner's problem[4],

$$P_{i,t} = \frac{\Lambda_{I,t}}{\Lambda_{c,t}} = \frac{\lambda_{c_2,t}}{\lambda_{c_1,t}} X_{2,t}^{\alpha-1} \tag{12}$$

In this economy, prices of investment good in physical capital and intangible capital are the same. This price has a downward trend as $\alpha - 1 < 0$.

Period t price of capital installed at beginning of period $t+1$, $P_{k'_x,t}$, equals to the inverse of the marginal adjustment cost with respect to the investment at date t,

$$P_{k'_x,t} = \frac{P_{ix,t}}{\phi'_x(\frac{I_{x,t}}{K_{x,t}})}, \text{ for } x = m, u$$

This is the fundamental relationship in the q theory. If firm only issues equity, $P_{k'_x}$ is the present value of the earnings of the underlying capital that equity is used to finance, hence has the interpretation of the price of a share of equity, as discussed in Christiano and Fisher (1998). The value of the market equity can be thought as total value of all the types of capital, that is,

$$P_{k',t} = \sum_{x=m,u} \frac{K_{x,t+1}}{K_{t+1}} P_{k'_x,t} \tag{13}$$

where $K_{t+1} = \sum_{x=m,u} K_{x,t+1}$.

Period t price of capital previously installed at beginning of period t is given by

$$P_{k_x,t} = [(1-\delta_x) + \phi_x(\frac{I_{x,t}}{K_{x,t}}) - \phi'_x(\frac{I_{x,t}}{K_{x,t}})\frac{I_{x,t}}{K_{x,t}}]P_{k'_x,t}, \text{ for } x = m, u \tag{14}$$

2.4.2. Market Value of Firms

Market value of firm equals to the market value of capital owned by this firm. Denote MV_t as the market value of the firms, then

$$MV_t = P_{k'_m,t}K_{m,t+1} + P_{k'_u,t}K_{u,t+1} = P_{i,t}\left[\frac{K_{m,t+1}}{\phi'_m(\frac{I_{mt}}{K_{mt}})} + \frac{K_{u,t+1}}{\phi'_u(\frac{I_{ut}}{K_{ut}})}\right]$$

Firm finances the capital by issuing bonds and equity, hence the market value of firm also equals the market value of equity and bonds issued by this firm

$$MV_t = ME_t + MB_t$$

where ME and MB are the market value of equity and market value of bonds, respectively.

The book value of firm (BV_t) is book value of total capital owned by this firm, which is same as the replacement cost of the total capital, that is

$$BV_t = P_{i,t}(K_{m,t+1} + K_{u,t+1})$$

which also equals to the sum of the book value of equity and market value of bonds,

$$BV_t = BE_t + BB_t$$

where BE and BB are the market value of equity and market value of bonds, respectively.

2.4.3. Tobin's q

Tobin's q is the ratio of price of installed capital goods to the price of investment goods. For type x capital, it is

$$q_{x,t} = \frac{P_{k'_x.t}}{P_{i_x,t}} = \frac{1}{\phi'_x(\frac{I_{xt,}}{K_{xt,}})} \text{ for } x = m, u \tag{15}$$

It is easy to verify that the aggregate Tobin's q is the weighted average of the Tobin's q of type x capital, weighted by the relative amount of the type x capital stock,

$$q_t = \frac{1}{K_{t+1}} \left[\frac{K_{m,t+1}}{\phi'_m(\frac{I_{mt}}{K_{mt}})} + \frac{K_{u,t+1}}{\phi'_u(\frac{I_{ut}}{K_{ut}})} \right] \tag{16}$$

Since the value of installed capital goods is same as the value of firms, the aggregate Tobin's q is the ratio of market value of firms to the replacement cost of the investment goods. Note that this q is not what plotted in Figure 2, which plots the ratio of market value of total capital to the replacement cost of physical capital. Denote this observed 'q' by q^*, then

$$q_t^* = \frac{1}{K_{mt+1}} \left[\frac{K_{m,t+1}}{\phi'_m(\frac{I_{mt}}{K_{mt}})} + \frac{K_{u,t+1}}{\phi'_u(\frac{I_{ut}}{K_{ut}})} \right] = \frac{1}{\phi'_m(\frac{I_{mt}}{K_{mt}})} + \frac{K_{u,t+1}}{K_{m,t+1}} \frac{1}{\phi'_u(\frac{I_{ut}}{K_{ut}})} \tag{17}$$

The steady-state value of q is 1, while the steady-state value of q^* is $1 + \frac{\bar{K}_u}{\bar{K}_m} > 1$. From equation (17) we know that the variations in q^* is partly driven by the variation in the share of intangible capital in total capital, the more is the intangible capital relative to physical capital, the larger is q^*. Firms with high market-to-book value of capital (MV/BV) are those firms with high intangible capital. Hansen, Heaton, and Li (2005) identify firms with high intangible capital based on high market equity-to book equity (ME/BE). Note that market-to-book value of capital is not same as market equity-to book equity, unless firms only issue equity. Hansen, Heaton, and Li (2005)'s identification of firms is reasonable if the equity-to-debt ratio is stable over time. The relation between MV/BV and ME/BE is

$$q_t^* = \frac{MV_t}{BV_t} = \frac{ME_t}{BE_t} \cdot \frac{MV_t/ME_t}{BV_t/BE_t}$$

If $\dfrac{MV_t/ME_t}{BV_t/BE_t}$ is stable over time, then firms with high intangible capital can be identified by high market equity-to-book equity. Figure 3[5] shows that this is approximately true in aggregate for US nonfarm and nonfinancial corporate firms during postwar periods except for 1980s. The top panel of Figure 3 shows that for these firms, market equity-to-book equity tracks market-to-book value of capital pretty well, and the correlation coefficient of these two ratios 0.93 during the period of 1947Q1-2009Q2. The bottom panel plot the ratios of equity to total value of firms measured in market value and book value, and the ratio of these two ratios. It shows that the$\dfrac{MV_t/ME_t}{BV_t/BE_t}$ is not too far away from 1 and not very volatile for most of the periods. The average of this ratio is 1.2 and standard deviation is 0.11.

Adjustment cost is another source for the variation in q^*. High marginal adjustment cost could drive q as well as q^* below 1 and above 1. It is easy to check that the elasticity of investment with respect to Tobin's q of type x capital at the steady state is ξ_x. The elasticity of the aggregate investment to the aggregate Tobin's q is a weighted average of the elasticities of investment with respect to Tobin's q of each type of capital, that is, $\xi \equiv \dfrac{\partial \log I_t}{\partial \log q_t} = \left[\sum_{x=m,u} \dfrac{w_x}{\xi_x}\right]^{-1}$, where $w_x = \dfrac{K_x/I_x}{K/I}$ is the weight. If the elasticity of the investment to Tobin's q of both types of capital are the same, then the elasticity of the aggregate investment to the aggregate Tobin's q is simply $\xi = \xi_m = \xi_u$.

2.4.4. Rates of Return

The one-period risk-free interest rate, r_t^f equals to the reciprocal of the date t conditional expectation of stochastic discount factor $m_{t,t+1}$ as defined in Hansen and Jagannathan (1991), which is same as the intertemporal marginal rate of substitution,

$$\begin{aligned} m_{t,t+1} &= \beta(\frac{C_{t+1}}{C_t})^{-\gamma}(\frac{1-N_{t+1}}{1-N_t})^{\theta(1-\gamma)} \\ r_t^f &= 1/E_t\left[m_{t,t+1}\right] \end{aligned}$$

The unconditional mean of risk-free rate equals to

$$\begin{aligned} &E(r_t^f) = \frac{1}{\beta}\exp(E(m_{t,t+1}) - \frac{1}{2}var(m_{t,t+1})) \\ &= \frac{1}{\beta\exp(-\gamma g_c)}\exp(-\frac{1}{2}\left(\gamma^2\sigma^2_{\Delta c} + \theta^2(1-\gamma)^2\sigma^2_{\Delta n}\right)) \end{aligned} \tag{18}$$

where σ_c and $\sigma_{\Delta n}$ are the standard deviations of consumption growth and change in working hour, respectively.

The rate of return on the capital of type x in terms of the consumption goods is given by

$$1 + r_{x,t+1} = \frac{mpk_{x,t+1} + P_{k_x,t+1}}{P_{k'_x t}}$$

where $mpk_{x,t+1}$ denotes the marginal product of capital of type x. From the assumption of competitive equilibrium, we know that the marginal product of capital should equal across different sectors.

Return on the capital of sector j in terms of the consumption goods, is a weighted average of the returns on different types of capital used in that sector, the weight is determined by the relative value of each type of the capital.

$$1 + r_{j,t+1} = \frac{P_{k'_m,t}K_{jm,t+1}}{\bar{K}_{j,t+1}}r_{m,t+1} + \frac{P_{k'_u,t}K_{ju,t+1}}{\bar{K}_{j,t+1}}r_{u,t+1}$$

where

$$\bar{K}_{j,t+1} = P_{k'_m,t} K_{jm,t+1} + P_{k'_u,t} K_{ju,t+1}$$

and the overall return on capital in the market is

$$1 + r_{t+1} = \frac{P_{k'_m,t} K_{m,t+1}}{\bar{K}_{t+1}} r_{m,t+1} + \frac{P_{k'_u,t} K_{u,t+1}}{\bar{K}_{t+1}} r_{u,t+1}$$

where $\bar{K}_{t+1} = P_{k'_m,t} K_{m,t+1} + P_{k'_u,t} K_{u,t+1}$ is the total value of capital in the market.

In each sector, firms issue bonds and equity to finance the capital expenditure. It is straightforward to show that the rate of return on equity in sector j is,

$$1 + r^e_{j,t+1} = (1 + \frac{BE_{j,t}}{ME_{jt}})(1 + r_{j,t+1}) - \frac{BE_{j,t}}{ME_{j,t}}(1 + r^f_t)$$

where $\frac{BE_{j,t}}{ME_{j,t}}$ denotes the firm's debt-to-equity ratio in sector j, which is not determined in the equilibrium as is well known. Suppose the firm's debt-to-equity ratio is same across sectors, then the overall return on the equity of the market is simply given by

$$1 + r^e_{t+1} = (1 + \frac{BE_t}{ME_t})(1 + r_{t+1}) - \frac{BE_t}{ME_t}(1 + r^f_t)$$

2.5. Calibration and Parameter Estimation

There are 14 model parameters θ, γ, β, α, δ_m, δ_u, α_{1m}, α_{2m}, g_1, g_2, ξ, σ_1, σ_2 in the model, where the adjustment cost coefficients for physical and intangible capital are assumed to be the same to keep things as simple as possible. The first step is to follow the practice of the business cycle literature by choosing some parameters to fit stead state relations. For the other parameters, *a prior* knowledge is not available in the literature. The second step is to choose values for these parameters to maximize the model's ability to replicate a set of business cycle and asset pricing moments.

The first set of parameters that governs steady state and long-run properties of the model economy. These parameters do not significantly affect model dynamics and are estimated based on National Income and Product Account (NIPA) data or *a prior* knowledge from business cycle literature.

The growth rates of the technology shocks g_1 and g_2 are chosen to match the annual growth rates of consumption of 3.4% and private fixed nonresidential investment of 4.2%,

$$\begin{aligned} \frac{1}{1-\alpha} g_1 + \alpha g_2 &= g_c = 0.034 \\ \frac{1}{1-\alpha} g_1 + g_2 &= g_k = 0.042 \end{aligned}$$

which implies $g_1 = 1.89\%$, $g_2 = 1.25\%$.

The intertemporal elasticity of the substitution between consumption and leisure, $\theta = 2.16$ is chosen such that a percentage of working hour is 25%. The constant capital share in the production function α is 0.36, which is assumed to be same in the two sectors. The capital share of physical capital, $\alpha_{1m} = 0.23$ and $\alpha_{2m} = 0.20$, are chosen to match the average ratio of nominal private fixed nonresidential investment to nominal consumption of consumer nondurable and services, which is 0.19 for in US economy during 1947Q1-2009Q2. The risk aversion coefficient in the utility function is fixed at $\gamma = 5$ for the benchmark parameterization. As discussed later, capital stock estimates are not sensitive to the choice of gamma for the models considered in this paper. The quarterly depreciation rate of physical capital is fixed at $\delta_m = 0.025$ based on the data of NIPA capital stocks and investment of private fixed nonresidential assets, which is same as that of Hall (2001).

The remaining parameters β, ξ, δ_u, σ_1, σ_2 govern the business cycles and asset pricing properties of the model economy, but previous empirical work does not offer much guidance on the choice of these parameters. we choose them to maximize the model's ability to reproduce the moments of interest. First, given the value of risk aversion coefficient, mean and standard deviation of consumption growth and working hour, the discount rate β is chosen to match the unconditional mean of risk-free rate according to equation (18). The average quarterly return on 90-day Treasury bill rate from 1947 Q1 to 2004 Q1 is 0.26%, which implies $\beta = 0.99$. Next, the remaining four parameters are chosen to match the following four moments obtained from business cycle and asset market data[6]: 1) the standard deviation of investment growth, 2) the standard deviation of consumption growth 3) the correlation between growth of market value of firms and consumption growth 4) the correlation between investment good price growth and consumption growth:

$$\sigma(\Delta \log I_{m,t}) = 0.030,\ \ \sigma(\Delta \log C_t) = 0.0055 \tag{19}$$

$$\rho(\Delta \log C_t, \Delta \log P_{i,t}) = 0.15, \rho(\Delta \log C_t, \Delta \log V_t) = 0.23 \tag{20}$$

I could not find values of the parameters to replicate these moments exactly. Instead, for a given set of values for adjustment cost coefficient and depreciation rate, we choose values for σ_1, σ_2 to reproduce (19) exactly and then get as close as possible to (20) by choosing $b = (\xi$, $\delta_u)$ to minimize the distance between the moments implied by the model and the moments estimated from data (20). The distance is measured by

$$\min_b \chi(b) = [\hat{J}_T - f(b)]'[\hat{J}_T - f(b)]$$

where $\hat{J}_T$ is the vector of the remaining moments (20) to match, $f(b)$ is the vector of moments generated by the model, which can easily be computed from the linear model solution. The minimization is done by searching over a grid of values for $b : \xi = [0.10, \infty]$, and $\delta_u = [0, 0.03]$.

The parameter values that drive down χ to 0.0064 are:

$$\hat{\xi} = 0.45,\ \ \hat{\delta}_u = 0.005$$

and the corresponding estimates σ_1 and σ_2 are:

$$\hat{\sigma}_1 = 0.0048, \hat{\sigma}_2 = 0.078$$

The capital adjustment cost technology is specified by one single parameter, ξ, which is the elasticity of investment with respect to Tobin's q. The estimates of ξ that Abel (1980) and Eberly (1997) got in somewhat different models range from 0.27 to 0.52 and from 0.37 to 1.06, respectively. My estimate of this parameter lies within the range reported in the literature. The estimate of σ_1 is roughly same as the standard deviation of consumption growth and not sensitive to the specification of other parameters. The depreciation rate of intangible capital, the standard deviation of investment-specific technology shock and the adjustment cost coefficient are to some extent substitute in generating the moments 2)-4), that is, volatility of investment growth and the correlation between investment goods price, market value and consumption growth. Roughly speaking, with lower elasticity of investment with respect to Tobin's q, thus higher adjustment cost, σ_2 has to be higher and δ_u has to be lower.

Notice that we did not choose parameter values to match the mean and standard deviation of equity returns, because this type of RBC model predicts very low equity premium as discussed in Mehra and Prescott (1985) and section 4. The reason is that with constant risk aversion preferences, the model fails to provide enough volatility in the marginal rates of substitution. Since the estimation of intangible capital exploit the information on both asset market data and macroeconomic variable, the poor asset market predictions affects the estimation of intangible capital, as discussed in section 4. Hence, better estimation of intangible requires better model description of the economy. Jermann (1998) shows that it is promising to add habit formation along with adjustment cost in the real business cycle models. Tallarini (1998) argues that with non-expected utility preferences, it is possible to improve the model's asset market predictions without significantly affecting the relative variabilities and comovement of aggregate quantity variables. Christiano and Fisher (1998) show that a real business cycle model with investment-specific technological shock, adjustment cost and habit formation preferences does better in replicating the comovement of stock prices and aggregate variables as well as equity premium than existing models. Estimation of intangible in a two-sector model with habit formation or non-expected utility preferences added is left for future research.

We also followed Hamilton (1994) to used Maximum likelihood Estimation to estimate these parameters, implemented using DYNARE and the estimation results confirm the above choice of the parameter values.

3. Estimation of Intangible Capital

In section 2, the dynamics of the model economy is summarized in a state-space representation (11) In this section, we use Kalman filter to obtain estimates of the unobserved state variables using the data on the observed variables, and in particular, infer the value of intangible capital stock.

3.0.1. The State Space Presentation of the Model

The state space representation of the model (11) is summarized as following,

State Equation:

$$s_{t+1} = Fs_t + Gu_t + \Gamma\varepsilon_{t+1}$$

where

$$s_t = \begin{bmatrix} \log K_t & \log X_t \end{bmatrix}', \varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} & \varepsilon_{2,t} \end{bmatrix}', u_t = 1$$

$$F = \begin{bmatrix} M_1 & \overrightarrow{B_k} - M_1 \\ 0 & E_2 \end{bmatrix}, \; G = \begin{bmatrix} \overrightarrow{B_k}\; g + (E_2 - M_1)\log k \\ g \end{bmatrix}, \Gamma = \begin{bmatrix} 0_{2\times 2} \\ E_2 \end{bmatrix}$$

Observation Equation:

$$y_t = A'u_t + H's_t + w_t$$

$$y_t = \begin{bmatrix} \log C_t & \log I_{m,t} & \log P_{i,t} & \log WN_t & \log MV_t \end{bmatrix}'$$

$$A' = \begin{bmatrix} \log(c) - \pi_{ck}\log(k) \\ \log(i_m) - \pi_{imk}\log(k) \\ \log(p_i) - \pi_{pk}\log(k) \\ \log(wn) - \pi_{wnk}\log(k) \\ \log(mv) - \pi_{vk}\log(k) \end{bmatrix} \quad H' = \begin{bmatrix} \pi_{ck} & B_c - \pi_{ck}\overrightarrow{B_k} \\ \pi_{imk} & B_k - \pi_{imk}\overrightarrow{B_k} \\ \pi_{pk} & B_c - B_k - \pi_{pk}\overrightarrow{B_k} \\ \pi_{wnk} & B_c - \pi_{wn}\overrightarrow{B_k} \\ \pi_{vk} & B_c - \pi_{vk}\overrightarrow{B_k} \end{bmatrix}$$

where $\overrightarrow{B_k} = \begin{bmatrix} B_k' & B_k' \end{bmatrix}'$, $B_k = \begin{bmatrix} 1/(1-\alpha) & 1 \end{bmatrix}$, $B_c = \begin{bmatrix} 1/(1-\alpha) & \alpha \end{bmatrix}$ and E_2 is a 2×2 identity matrix. s_t is the vector of state variables (unobservable), and y_t is the vector of variables observed at date t. In the two-sector economy of this paper, s_t contains the logs of capital stock ($\log K_t$) and technology shock ($\log X_t$); y_t contains the logs of consumption, investment in physical capital, labor income, relative price of investment goods, and market value of firms. M_1, M_2 and π's are from the linear solution of the model (9) and (10). w_t is the vector of measurement errors associated with each observed variables, which is assumed to be vector of white noise and uncorrelated with ε_t at all lags, that is,

$$E[w_t w'_\tau] = \begin{cases} \Sigma_w & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases}$$
$$E[\varepsilon_t w'_\tau] = 0 \qquad \text{for all } t \text{ and } \tau.$$

where Σ_w is the variance-covariance matrix of the measurement error w_t. The measurement errors for each of the observable variables are assumed to be uncorrelated with each other and have same variance, that is, $\Sigma_w = \sigma_w^2 E_n$, where E_n an $n \times n$ identity matrix, n is the dimension of $y_{t.}$ The magnitude of standard deviation of measurement error σ_w is assumed to be small relative to the standard deviation of technology shocks[7].

Following Hamilton (1994), we use Kalman filter algorithm to recursively calculate the linear least square forecasts of the state vector s_t on the basis of data observed through date t, and more efficient estimates of the value of capital stock at date t can be estimated based on the full set of data collected by using the smoothed estimate of s_t

3.1. Estimation Results

In this subsection, we estimate the intangible capital in the two-sector model using quarterly U.S. data from 1947Q1 to 2009Q2. The vector of observable y_t consists of consumption, investment, labor income, investment good prices, as well as the market value of capital of nonfarm and nonfinancial firms. Most of macroeconomic time series are from NIPA. Consumption is the sum of consumption of consumer nondurable goods and services. The consumption deflator is imputed from the price deflator of nondurables and services. Investment is the private fixed nonresidential investment from Flow Funds Accounts of Federal Reserve Board. The market value of capital is market value of equity plus market of bonds of nonfarm and nonfinancial corporate firms minus value of inventory, constructed in the same way as Hall (2001). For comparison, we also presents the estimation results in Hall (2001)'s one-sector model using the approach developed in this paper.

3.1.1. One-Sector Model of Hall (2001)

In Hall (2001), intangible capital stock is imputed as the difference between total capital stock and physical capital stock. Total capital stock in the economy is imputed from following equation

$$\frac{MV_t}{K_{t+1}} = q_t = \left[1 + \phi(\frac{K_{t+1} - (1-\delta)K_t}{K_t})\right]$$

where MV_t is market value of capital, which is assumed to be same as the market value of nonfarm and nonfinancial corporate firms, K_t is the sum of physical and intangible capital stock. ϕ is the coefficient of the quadratic adjustment cost function, Hall (2001) chooses $\phi = 8$ for fast adjustment and $\phi = 32$ for slow adjustment[8]. Physical capital stock $(K_{m,t})$ is calculate using the following capital accumulation rule,

$$K_{m,t} = (1-\delta)K_{m,t-1} + I_{m,t}$$

where depreciation rate is taken to be 10% per year, $I_{m,t}$ is private fixed nonresidential investment from Flow Funds Accounts of Federal Reserve Board.

we rewrite Hall's economy as a one-sector economy, tangible and intangible capital are perfect substitutes in this economy. The total capital stock is estimated using Kalman filter algorithm described in the above subsection. Key parameter values are the same as that of Hall. Market value of the equity and bonds (MV_t) and real consumption (C_t) data is used to imputed capital stock. Figure 4 presents the estimates of investment and capital stock of total capital in this economy along with the estimates capital of Hall (2001). The estimates of total capital are almost same as Hall (2001)'s estimates except for the initial periods. The difference is due the log-linearizing error. The implied investment of capital is very volatile as compared with investment in physical capital. Figure 5 presents the results with real consumption growth added as additional input in Kalman filter, estimates of intangible capital are much smaller and much smoother than the estimates using only market value of capital. Figure 6 shows that the estimation results are poor in this case, and the model could not fit well consumption and market value of capital simultaneously. As compared with data, the model implied consumption is too volatile and market value is too smooth. The reason behind this result is that, with adjustment cost added to the one-sector standard RBC model, consumption is forced to absorb extra volatility as investment fluctuations are costly. With constant risk aversion and adjustment cost, this type of model implied very high volatility of consumption and fails to match the data on the volatility of consumption and of market value of capital at same time[9].

Hall (2001) argues that U.S. corporation have formed large amount of intangible capital over past 50 years, especially in 1990s, this is shown in Figure 4, where the difference between total capital estimated in the model and physical capital is intangible capital. For fast capital adjustment, intangible capital inferred from market value of corporate firms rises dramatically in the last decade. However, my calculation shows that this result hinges on the choice of market value of capital as the only observable variable inputs. If real consumption is added as input as well, inferred intangible capital is very stable.

3.1.2. The Two-Sector Model with Investment-Specific Technological Progress

For the two-sector model developed in this paper, we first estimate capital stock using only macroeconomic aggregate variable, then add market value of total capital to the observable variables as the Kalman filter inputs.

Figure 7 and Figure 8 presents the estimates using only macroeconomic aggregate variable. The top panel of Figure 7 plots data on private fixed nonresidential investment ($dataIm$), estimates of investment in physical capital (Im) and intangible capital (Iu) and The bottom panel presents the estimate of physical capital (Km), intangible capital (Ku)

and the physical capital stock ($dataKm$) calculated using capital accumulation rule (6) and data on private fixed nonresidential investment with the adjustment cost coefficient and depreciate rate specified same as the model estimates ($\xi_m = 0.45$ and $\delta_m = 0.025$). All the time series are reported in logs. The estimates of investment of physical capital investment is relatively smooth as compared with data. The growth rate and volatility of intangible capital are about the same as that of physical capital. The growth rate of physical stock estimated using Kalman filter is less than that of the capital stock calculated based primarily on investment. The top panel of Figure 8 shows that the estimates of market value of capital has less variation than data. The bottom panel show that the implied market-to-book value (q^*) matches the data much better than the estimates from Hall (2001) economy.

Figure 10 and Figure 11 present estimates using both data on macroeconomic aggregate variables and market value of capital. Figure 10 shows that adding market value of capital drives up the growth rate as well as the volatility of investment, while Figure 11 indicates that the estimates of market value of capital is improved and the implied q^* is larger than the estimates without using data on market value of capital. than the real data. This model implies a much higher variability in market-to-book ratio than Hall (2001)'s models, while keeps the variability of investment reasonable.

To access the accuracy of the measurement, we calculate the ratio of conditional variance to the conditional second moment of capital stock $\frac{\sigma_t^2(\log k_x)}{E_t[(\log k_x)^2]}$, given the current and past data on observable variables. $\sigma_t^2(\log k_x)$ is the conditional variances of capital k_x, which can be computed from the MSE of the state variables (**??**), and $E_t[(\log k_x)^2] = \sigma_t^2(\log k_x) + (E_t[(\log k_x)])^2$ is the conditional second moment of capital stock k_x, where $E_t[(\log k_x)]$ can be computed from the Kalman filter estimates of the state variables (**??**). The ratio $\frac{\sigma_t^2(\log k_x)}{E_t[(\log k_x)^2]}$ lies between 0 and 1, and closer it is to 0, more accurate are the estimates. The estimates of the capital stock are pretty accurate, as shown bottom panel of in Figure 12, which depicts the ratio $\frac{\sigma_t^2(\log k_x)}{E_t[(\log k_x)^2]}$ for physical and intangible capital. For both physical capital and intangible capital, the ratios are very close to zero for most of the periods. The top panel of Figure 12 depicts the conditional standard deviation of estimates of physical and intangible capital given current and past data. Dash line and solid line plot the estimates with and without using market value of capital, respectively. It shows that the accuracy of measurement is improved by 50% with market value of capital added in the estimation for both types of capital.

4. Implications of the Model

Before concluding the paper, we would like to discuss some of the asset market and macroeconomic implication of the model. In the two-sector model developed in this paper, three important elements are added to the standard RBC model, capital adjustment cost, intangible capital, and investment-specific technology shock.

Capital adjustment costs drive the wedge between the marginal cost of installed capital and that of new capital goods, which are measured by stock prices and investment good prices, respectively. Adding adjustment cost to the business model helps to deliver variation in q. However, with modest adjustment cost, the implied q can not be too far away from its steady-state value of 1. Adding intangible capital to the model helps to explain the difference between the market value of total capital and replacement cost of physical capital. As shown in section 2.4.3., the steady-state value of q^* could be larger than 1. Table 1 summarizes the volatility of q^* , q and capital stock implied in the two-sector model with various specification of adjustment cost. For the benchmark specification of adjustment cost, $\xi = 0.45$, adjustment cost accounts for about 56% of the variation in the estimates of q^* and this percentage ranges from 56% to 72% for different specifications of ξ. Table 1 also presents the variation of capital stock implied by the model for different values of ξ while holding other parameter values unchanged[10]. The volatility of capital stock rises while the variation of q^* decrease with the elasticity of investment to Tobin's q. Figure 13 plots the estimates of q^*, logs of physical and intangible capital for different values of ξ. The growth rate of implied capital stock is not

Table 1: Volatility of Market-to-Book Ratio

	$\xi = 0.225$	$\xi = 0.45$	$\xi = 4.5$	$\xi = \infty$
$\sigma(q^*)\%$ (data)	52.09	52.09	52.09	52.09
$\sigma(q^*)\%$	54.47	38.73	5.00	0.94
$\sigma(q)/\sigma(q^*)$	0.57	0.56	0.72	0.69
$\sigma(K_m)\%$	0.98	0.97	3.17	3.83
$\sigma(K_u)\%$	1.11	1.11	3.49	4.51

sensitive to the specification of ξ, while the volatility is higher when it is less costly to adjust capital stock. The accuracy of measurement decreases with the elasticity of investment to Tobin's q or increases with adjustment cost, as shown in Figure 13 left top panel, where the norm of conditional variance-covariance matrix of estimates of state variables is plotted.

On the other hand, capital adjustment penalize investment variation and forces consumption to absorb the extra volatility, hence simply adding adjustment cost in the standard RBC model implies too much consumption volatility and too little investment volatility, as shown in Table 2, for the one-sector model of Hall (2001). In this model, the relative volatility of investment growth to the volatility of consumption growth is much lower than that of data. In addition, one-sector models imply too little variation in the market value of capital and equity returns. In the two-sector model developed in this paper, by adding investment specific shock, the model can match the volatility of consumption and investment at same time. In addition, as shown in Table 2, the model implied volatility of market value of capital and equity returns is improved but still has a long way to match the data. However, both models produce too little mean risk premium, because the constant risk aversion preference fails to provide enough volatility in the intertemporal marginal rate of substitution.

Since the estimation of intangible capital exploit the information on both asset market data and macroeconomic variable, the poor asset market predictions may affect the estimation of intangible capital. Hence, better estimation of intangible requires better model description of the economy. Jermann (1998) shows that it is promising to add habit formation along with adjustment cost in the real business cycle models. Tallarini (1998) argues

Table 2: Relative Volatility of Aggregate Varibles and Asset Returns

Model Version	$\sigma(\Delta c)$	$\frac{\sigma(\Delta I_m)}{\sigma(\Delta c)}$	$\frac{\sigma(\Delta I_u)}{\sigma(\Delta c)}$	$\frac{\sigma(\Delta mv)}{\sigma(\Delta c)}$	$E(r_f)$	$E(r-r_f)$	$\sigma(r_f)$	$\sigma(r)$
One-Sector Model	2.20	1.11	n/a	0.27	1.16	0.00	0.05	0.12
Two-Sector Model	2.20	5.16	5.02	2.47	0.56	0.28	1.20	5.64
Data	2.20	5.07	n/a	12.76	1.16	6.97	2.92	33.05

that with recursive utility preferences, it is possible to improve the model's asset market predictions without significantly affecting the relative variabilities and comovement of aggregate quantity variables. Christiano and Fisher (1998) show that a real business cycle model with investment-specific technological shock, adjustment cost and habit formation preferences does better in replicating the comovement of stock prices and aggregate variables as well as equity premium than existing models. Estimation of intangible capital in a two-sector model with recursive utility preferences is left for future research..

5. Conclusion

In this paper we have developed a two-sector model with intangible capital and investment-specific technological progress, and estimated the time series of the intangible capital accumulated in the US economy during the postwar period. For comparison, the value of the intangible capital is also inferred in the models used by Hall (2001). Since the time series of both macroeconomic variables and asset prices are used to make inferences about the intangible capital, it is important to develop a model that is consistent with the salient facts in both macroeconomics and asset markets. The two-sector model developed in this paper does better in both dimensions than the other two models, hence provide a better framework to make inference about the intangible capital accumulate in US economy.

The inference of the intangible capital depends on the specification of the capital adjustment costs. With modest capital adjustment costs, all of the three models implies that significant amount of the intangible capital has been accumulated during the past 50 years in the US economy. Hall (2001) argues the accumulation of intangible capital is much faster in the last decade as reflected in the stock market, while the two-sector model developed in this paper suggests that the growth rate of intangible capital during the 1990s is much lower than the estimates in Hall (2001).

For the two-sector economy with investment-specific technological progress, variation in intangible capital estimated from aggregate macroeconomic variables, accounts for almost half of the variability in the market-to-book ratio of nonfinancial and nonfarm corporate firms, while in the other two models the explained variability of the market-to-book ratio is much smaller. Firms with large amount of the intangible capital are the firms with high market-to-book value of capital. If the leverages of the firms are stable over time, these firms also have high market equity-to-book equity ratios. The result implies that the risk of the intangible capital explains part of risk identified by the market equity-to-book equity portfolios. Since the estimation exploits the information on the investment goods prices, the risk of intangible capital is partly caused by the investment-specific technological shock.

The two-sector model developed in this paper is able to produce significant variation in the market value of firms, but its implied risk premium is still much lower than the data. As showed in Jermann (1998) and Tallarini (1998), adding habit formation or non-expected utility preferences may help the current model to generate higher risk premium. Future research can be done to estimate the intangible capital by exploiting the information on the asset returns in such models.

Notes

[1]Investment is private fixed nonresidential investment, taken from Flow Funds Accounts of Federal Reserve Board. Data on consumption and price deflator is taken from NIPA.

[2]The formulas are:

$$a_{1,x} = (\exp(g_{kx}) - 1 + \delta_x)^{1/\xi_x}, \qquad a_{2x} = \frac{1}{\xi_x - 1}(1 - \delta_x - \exp(g_{kx}))$$

where g_{kx} is the mean growth rate of capital x, for $x = m, u$.

[3]See section 3 for details.

[4]See Appendix A for the details.

[5]Market value of equity and book value of bond for nonfarm and nonfinancial corporate firms are from Federal Resever Board Flow Funds Accounts. Book value of equity is calculated as the replacement cost of plant and equipment minus book value of bonds. For details of the data construction and calculation of market value of bonds, see data appendix of Hall (2001).

[6]Quarterly data on consumption and price deflators is taken from NIPA, quarterly data on investment and market value is taken from Flow Funds Accounts maintained by Federal Reserve Board. Consumption is nominal consumption of nondurable and services deflated by implicit price deflator of consumer nondurables and services, and investment is nominal private fixed nonresidential investment deflated by implicit price deflator for private fixed nonresidential investment. Investment goods price is implicit price deflator of private fixed nonresidential investment. Market value of nonfinancial and nonfarm corporate business is the sum of market value of equity and debt of these firms as described in Hall (2001).

[7]Practically σ_w is chosen to be $10^{-3}\sigma_1$, where σ_1 is the standard deviation of common technology shock to both of the two sectors and is smaller than $\sigma_{2,}$ the standard deviation of investment specific techonology shock. My explorations suggest that the results are not

sensitive on the specification measurement error as long as it is relatively small as compared with the standard deviation of techonology shocks.

[8]Hall (2001) also assumes asymmetric adjustment cost, but my calculation shows that the estimates change very little without this assumption.

[9]See section 4 for more discussion.

[10]My explorations suggest that the model estimates are not sensitive to the specification of the depreciation rate, curvature of utility function and the standard deviation of technology shocks.

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APPENDIX

A MODEL SOLUTION

A1. The Planner's Problem

The social planner's problem is to maximize the utility representative agent subject to resource constraints and capital accumulation equation. The Lagrangian equation for this problem is

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \{ \frac{[C_t(1-N_{1t}-N_{2t})^{\theta}]^{1-\gamma}}{1-\gamma}$$
$$+\Lambda_{c_1,t}[X_{1,t}K_{1m,t}^{\alpha_{1m}}K_{1u,t}^{\alpha_{1u}}N_{1,t}^{1-\alpha_{1m}-\alpha_{1u}} - C_t]$$
$$+\Lambda_{c_2,t}[X_{1,t}K_{2m,t}^{\alpha_{2m}}K_{2u,t}^{\alpha_{2u}}(X_{2,t}N_{2,t})^{1-\alpha_{2m}-\alpha_{2u}} - I_{mt} - I_{ut}]$$
$$+\Lambda_{i_m,t}[(1-\delta_m)K_{m,t} + \phi_m(\frac{I_{m,t}}{K_{m,t}})K_{m,t} - K_{m,t+1}]$$
$$+\Lambda_{i_u,t}[(1-\delta_u)K_{u,t} + \phi_u(\frac{I_{u,t}}{K_{u,t}})K_{u,t} - K_{u,t+1}]$$
$$+\Lambda_{m,t}[K_{m,t} - K_{1m,t} - K_{2m,t}]$$
$$+\Lambda_{u,t}[K_{u,t} - K_{1u,t} - K_{2u,t}]$$

where

$$\begin{aligned} \log X_t &= g + \log X_{t-1} + \varepsilon_t,\ \varepsilon_t \sim N\begin{pmatrix} 0 & & \sigma_1^2 & 0 \\ & , & & \\ 0 & & 0 & \sigma_2^2 \end{pmatrix} \\ \log X_t &= [\log X_{1t}, \log X_{2t}]',\ \ g = [g_1, g_2]' \end{aligned}$$

and $\Lambda_{x,t}$ are Lagrangian multipliers, for $x = c_1, c_2, i_m, i_u, m, u$. The functions of adjustment cost are given by

$$\phi_x(\frac{i_{x,t}}{k_{x,t}}) = \frac{a_{x,1}}{1-1/\xi_x}\left(\frac{i_{x,t}}{k_{x,t}}\right)^{1-1/\xi_x} + a_{x,2}$$
$$\phi'_x(\frac{i_{x,t}}{k_{x,t}}) = a_{x,1}\left(\frac{i_{x,t}}{k_{x,t}}\right)^{-1/\xi_x}, \ x = m, u$$

where $a_{1,x} = (\exp(g_k) - (1-\delta_x))^{1/\xi_x}$ and $a_{2x} = \frac{-1}{\xi_x - 1}(\exp(g_k) - (1-\delta_x))$ are set to make the steady-state growth path invariant to ξ_x for $x = m, u$.

A2. Transformed Economy

To solve the planner problem, first we need to transform the economy to a stationary system. The transformation involves dividing all variables in this economy by their stochastic trends except for the labor inputs. The variables in the transformed problem are c_t, $n_{1,t}$, $n_{2,t}$, $i_{m,t}$, $i_{u,t}$, $k_{m,t+1}$, $k_{u,t+1}$, $\lambda_{c_1,t}$, $\lambda_{c_2,t}$, $\lambda_{iu,t}$, $\lambda_{im,t}$, $\lambda_{m,t}$, $\lambda_{u,t}$. These are defined as follows,

$$c_t = \frac{C_t}{X_{ct}},$$

$$n_{1,t} = N_{1,t},\ n_{2,t} = N_{2,t}$$

$$k_{m,t+1} = \frac{K_{m,t+1}}{X_{kt}},\ k_{u,t+1} = \frac{K_{u,t+1}}{X_{kt}}$$

$$k_{jm,t} = \frac{K_{jm,t}}{X_{kt}},\ k_{ju,t} = \frac{K_{ju,t}}{X_{kt}},\ j = 1, 2$$

$$i_{m,t} = \frac{I_{m,t}}{X_{kt}},\ i_{u,t} = \frac{I_{u,t}}{X_{kt}}$$

$$\lambda_{c_1,t} = \Lambda_{c_1,t}X_{ct}^{\gamma},\ \lambda_{c_2,t} = \Lambda_{c_2,t}\frac{X_{kt}}{X_{ct}^{1-\gamma}},\ \lambda_{i_m,t} = \Lambda_{i_m,t}\frac{X_{kt}}{X_{ct}^{1-\gamma}}$$

$$\lambda_{i_u,t} = \Lambda_{i_u,t}\frac{X_{kt}}{X_{ct}^{1-\gamma}}, \lambda_{m,t} = \Lambda_{m,t}\frac{X_{kt}}{X_{ct}^{1-\gamma}}, \lambda_{u,t} = \Lambda_{u,t}\frac{X_{kt}}{X_{ct}^{1-\gamma}}$$

where $\alpha = \alpha_{1m} + \alpha_{1u} = \alpha_{2m} + \alpha_{2u}$, and

$$\log X_{ct} = \frac{1}{1-\alpha} \log X_{1t} + \alpha \log X_{2t} \equiv B_c \log X_t$$

$$\log X_{kt} = \frac{1}{1-\alpha} \log X_{1t} + \log X_{2t} \equiv B_k \log X_t$$

Transformed Lagrangian is then given by

$$\begin{aligned}
&\max E_0 \sum_{t=0}^{\infty} \beta^{*t} \exp((1-\gamma) \sum_{s=0}^{t} \varepsilon_{c,s}) \{ \frac{[c_t(1-n_{1t}-n_{2t})^{\theta}]^{1-\gamma}}{1-\gamma} \\
&+\lambda_{c_1,t}[k_{1m,t}^{\alpha_{1m}} k_{1u,t}^{\alpha_{1u}} n_{1,t}^{1-\alpha_{1m}-\alpha_{1u}} - c_t] \\
&+\lambda_{c_2,t}[k_{2m,t}^{\alpha_{2m}} k_{2u,t}^{\alpha_{2u}} N_{2,t}^{1-\alpha_{2m}-\alpha_{2u}} - i_{mt} - i_{ut}] \\
&+\lambda_{i_m,t}[(1-\delta_m)k_{m,t} + \phi_m(\frac{i_{m,t}}{k_{m,t}})k_{m,t} - \exp(g_k + \varepsilon_{k,t+1})k_{m,t+1}] \\
&+\lambda_{i_u,t}[(1-\delta_u)k_{u,t} + \phi_u(\frac{i_{u,t}}{k_{u,t}})k_{u,t} - \exp(g_k + \varepsilon_{k,t+1})k_{u,t+1}] \\
&+\lambda_{m,t}[k_{m,t} - k_{1m,t} - k_{2m,t}] \\
&+\lambda_{u,t}[k_{u,t} - k_{1u,t} - k_{2u,t}]
\end{aligned}$$

where $\beta^* = \beta \exp((1-\gamma)g_c)$, $g_c = B_c g$.

A3. First-Order Condition

The first order conditions for an interior solution to the stationary version of the planner problem can be rearranged to form the following system of equations:

$$[c_t] : 0 = c_t^{-\gamma}(1-n_{1t}-n_{2t})^{\theta(1-\gamma)} - \lambda_{c_1,t}$$

$$[n_{j,t}] : 0 = -\theta c_t^{1-\gamma}(1-n_{1t}-n_{2t})^{\theta(1-\gamma)-1} + \lambda_{c_j,t}(1-\alpha_j)k_{jm,t}^{\alpha_{jm}} k_{ju,t}^{\alpha_{ju}} n_{jt}^{-\alpha_j} \quad j = 1, 2$$

$$[i_{m,t}] : 0 = \lambda_{i_m,t}\phi_m'(\frac{i_{m,t}}{k_{m,t}}) - \lambda_{c_2,t}$$

$$[i_{u,t}] : 0 = \lambda_{i_u,t}\phi'_u(\frac{i_{u,t}}{k_{u,t}}) - \lambda_{c_2,t}$$

$$[k_{jm,t}] : 0 = \alpha_{jm} k_{jm,t}^{\alpha_{jm}-1} k_{ju,t}^{\alpha_{ju}} n_{jt}^{1-\alpha_j} \lambda_{c_j,t} - \lambda_{m,t},\ j = 1, 2$$

$$[k_{ju,t}] : 0 = \alpha_{ju} k_{jm,t}^{\alpha_{jm}} k_{ju,t}^{\alpha_{ju}-1} n_{jt}^{1-\alpha_j} \lambda_{c_j,t} - \lambda_{u,t},\ j = 1, 2$$

$$\begin{aligned}[k_{x,t}] \quad : \quad & 0 = -\lambda_{i_x,t}\exp(g_k + \varepsilon_{k,t+1}) + \beta^* E_t[\exp((1-\gamma)\varepsilon_{c,t+1})\{\lambda_{x,t+1} \\ & +\lambda_{i_x,t+1}\left[(1-\delta_x) + \phi_x(\frac{i_{x,t+1}}{k_{x,t+1}}) - \frac{i_{x,t+1}}{k_{x,t+1}}\phi'_x(\frac{i_{x,t+1}}{k_{x,t+1}})\right]\end{aligned}$$

where $g_x = B_x g$, $\varepsilon_{x,t+1} = B_x \varepsilon_{t+1}$ for $x = c, k$.

The relative prices studied in the main text can be derived using the multiplier from the above solution. First the relative price of the new investment good is given by

$$P_{i,t} = \frac{\Lambda_{c_2,t}}{\Lambda_{c_1,t}} = \frac{\lambda_{c_2,t}}{\lambda_{c_1,t}}\frac{X_{c,t}}{X_{k,t}} = \frac{\lambda_{c_2,t}}{\lambda_{c_1,t}} X_{2,t}^{\alpha-1}$$

Second, the prices for $K_{m,t+1}$ and $K_{u,t+1}$ are

$$P_{k'_x,t} = \frac{\Lambda_{i_x,t}}{\Lambda_{c_1,t}} = \frac{\lambda_{i_x,t}}{\lambda_{c_1,t}}\frac{X_{c,t}}{X_{k,t}} = \frac{P_{i,t}}{\phi'_x(\frac{I_{x,t}}{K_{x,t}})},\ \text{for } x = m, u$$

Third the prices for the installed capital are

$$P_{k_x,t} = P_{k'_x,t}[(1-\delta_x) + \phi_x(\frac{I_{x,t}}{K_{x,t}}) - \frac{I_{x,t}}{K_{x,t}}\phi'_x(\frac{I_{x,t}}{K_{x,t}})]$$

Notice that each of these prices has a stochastic trend of $X_{2,t}^{\alpha-1}$, which is a negative trend if $X_{2,t}$ trends upward.

A4. Steady-State Growth Path

The steady state of the transformed economy is characterized by the following equations.

$$\frac{i_x}{k_x} = \exp(g_k) - (1-\delta), \text{ for } x = m, u$$

$$c = k_{1m}^{\alpha_{1m}} k_{1u}^{\alpha_{1u}} n_1^{1-\alpha_{1m}-\alpha_{1u}}$$

$$i_m + i_u = k_{2m}^{\alpha_{2m}} k_{2u}^{\alpha_{2u}} N_2^{1-\alpha_{2m}-\alpha_{2u}}$$

$$k_x = k_{1x} - k_{2x}, \text{ for } x = m, u$$

$$\phi_x(\frac{i_x}{k_x}) = \exp(g_k) - (1-\delta_x), \phi_x'(\frac{i_x}{k_x}) = 1$$

$$0 = c^{-\gamma}(1-n_1-n_2)^{\theta(1-\gamma)} - \lambda_{c1}$$

$$0 = -\theta c^{1-\gamma}(1-n_1-n_2)^{\theta(1-\gamma)-1} + \lambda_{cj}(1-\alpha_j) k_{jm}^{\alpha_{jm}} k_{ju}^{\alpha_{ju}} n_j^{-\alpha_j}$$

$$0 = \lambda_{im} - \lambda_{c_2} = \lambda_{iu} - \lambda_{c_2}$$

$$0 = \alpha_{jm} k_{jm}^{\alpha_{jm}-1} k_{ju}^{\alpha_{ju}} n_j^{1-\alpha_j} \lambda_{c_j} - \lambda_m, \; j = 1, 2$$

$$0 = \alpha_{ju} k_{jm}^{\alpha_{jm}} k_{ju}^{\alpha_{ju}-1} n_j^{1-\alpha_j} \lambda_{c_j} - \lambda_u, \; j = 1, 2$$

$$0 = -\lambda_{ix} \exp(g_k) + \beta^* \{\lambda_x + \lambda_{ix} \left[(1-\delta_x) + \frac{1}{\xi_x}(\exp(g_k) - (1-\delta_x)) \right], x = m, u$$

B FIGURES

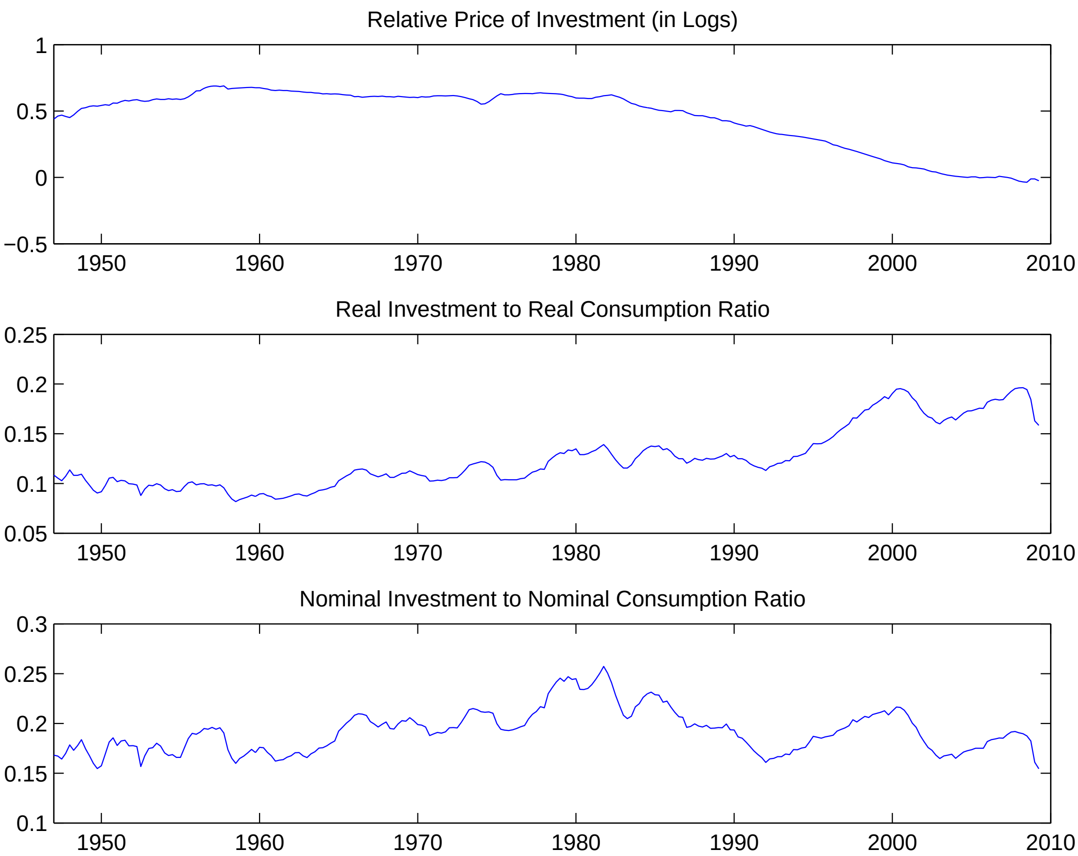

Figure 1: Relative Prices and Quantities of Private Fixed Nonresidential Investment to Consumption of Nondurables and Services in US Economy (1947Q1-2009Q2)

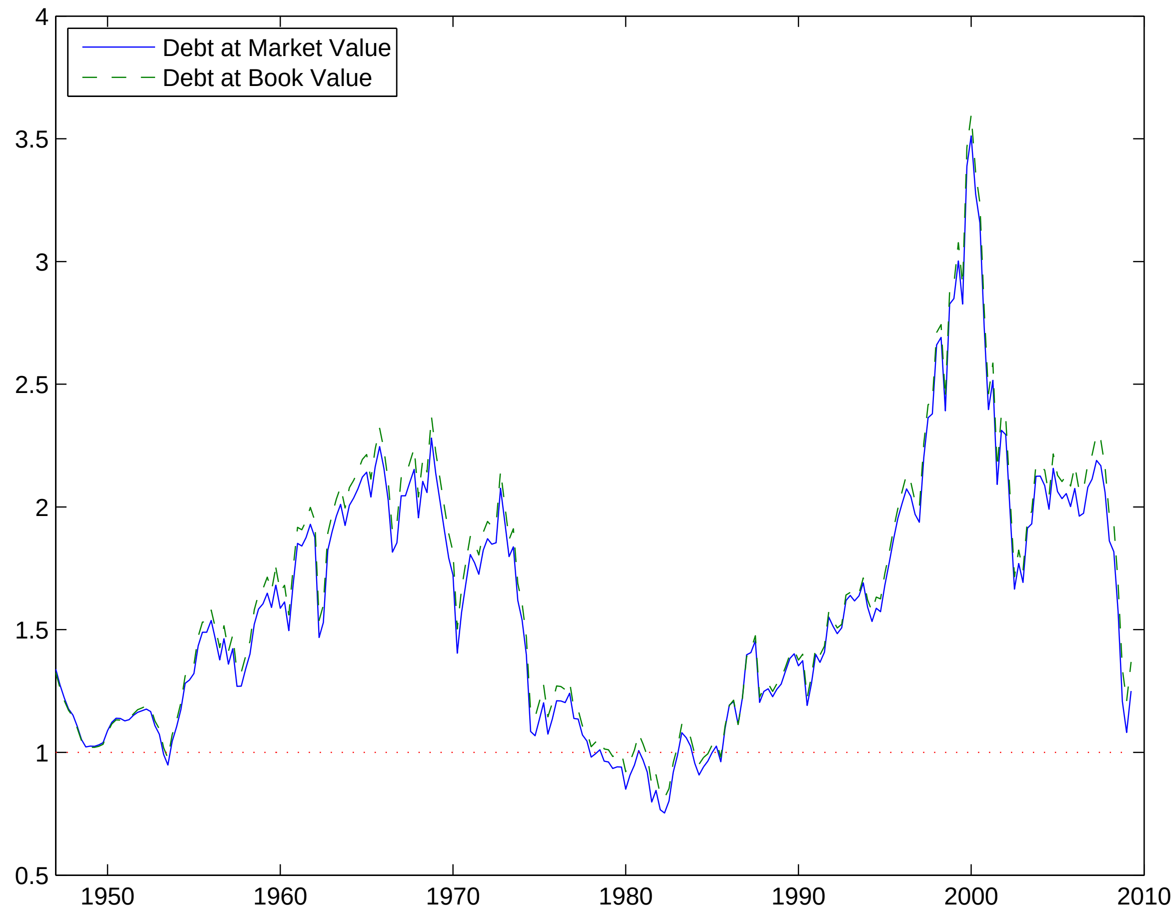

Figure 2: Ratio of Market Value to Replacement Cost of Plant and Equipment in US Economy

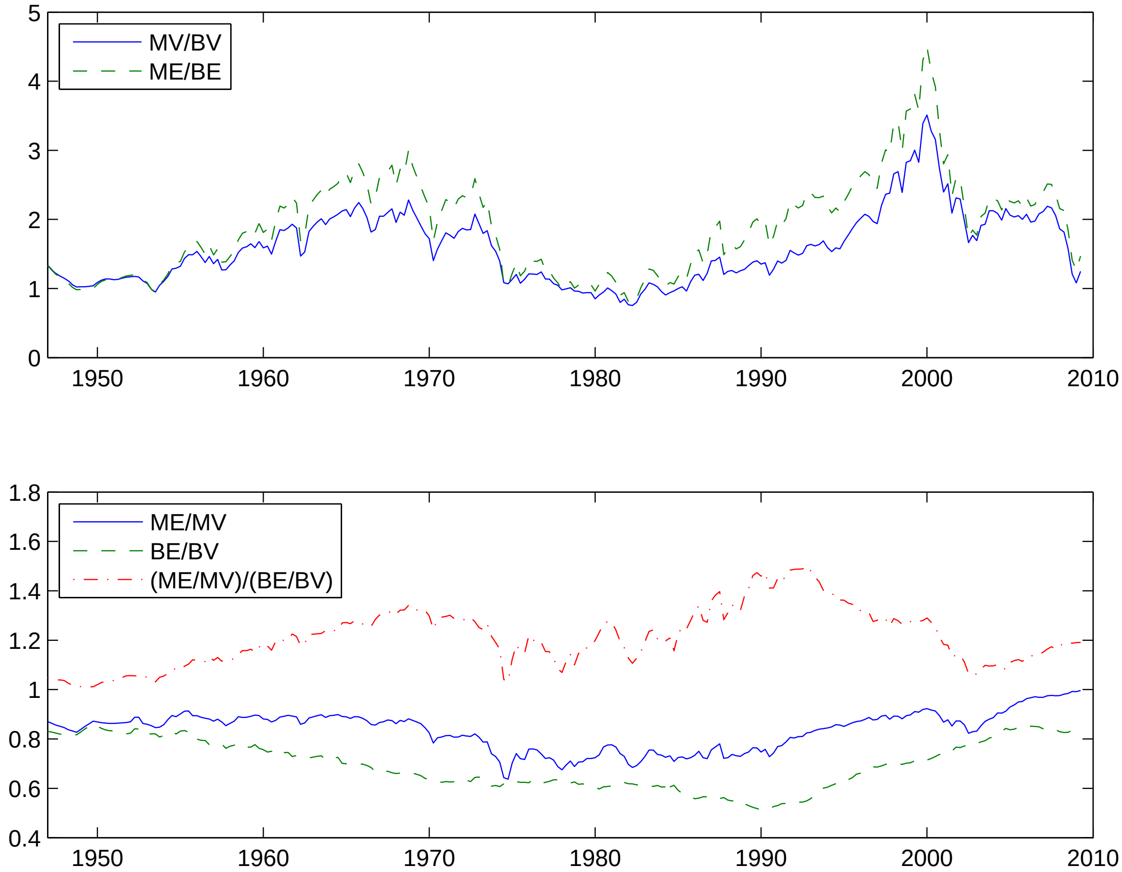

Figure 3: Relation between Market-to-Book Value of Capital and Market Equity-to-Book Equity for Nonfarm Nonfinancial Corporate Firms

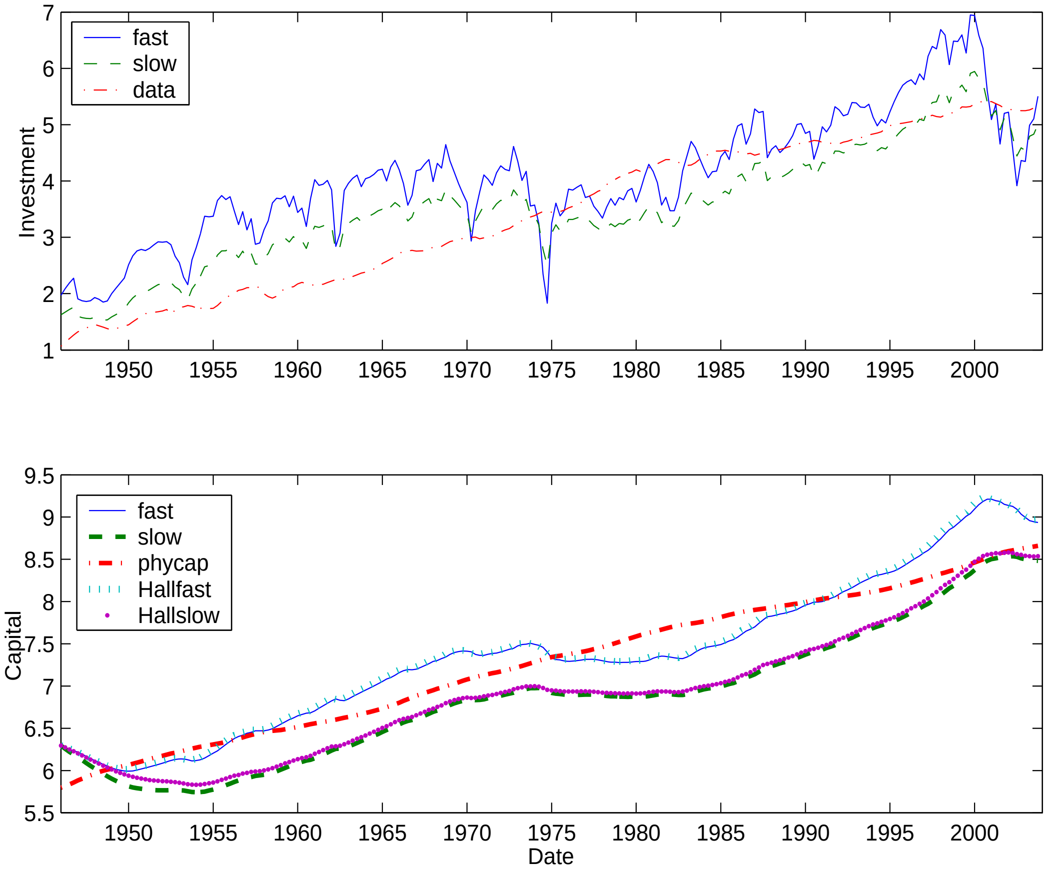

Figure 4: Investment and Capital Imputed from Market Value of Capital in the One-Sector Model of Hall (2001)

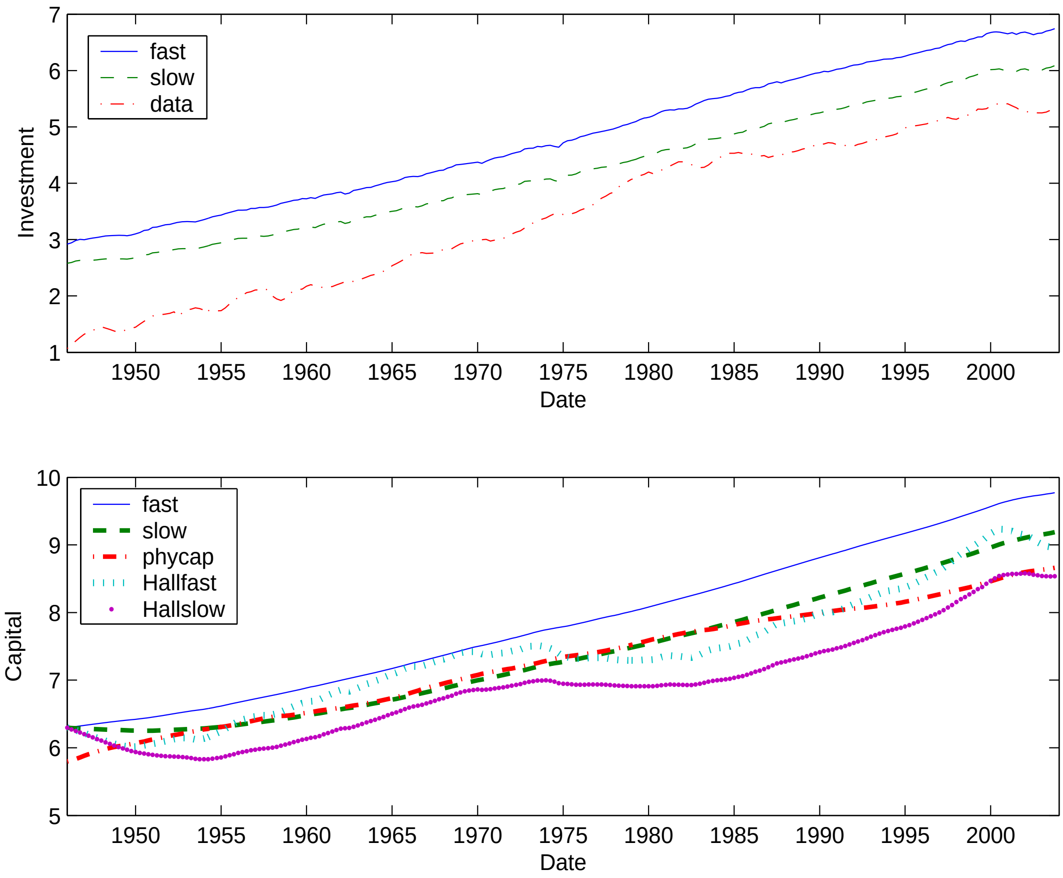

Figure 5: Investment and Capital Imputed from Market Value of Capital and Consumption in the One-Sector Model of Hall (2001)

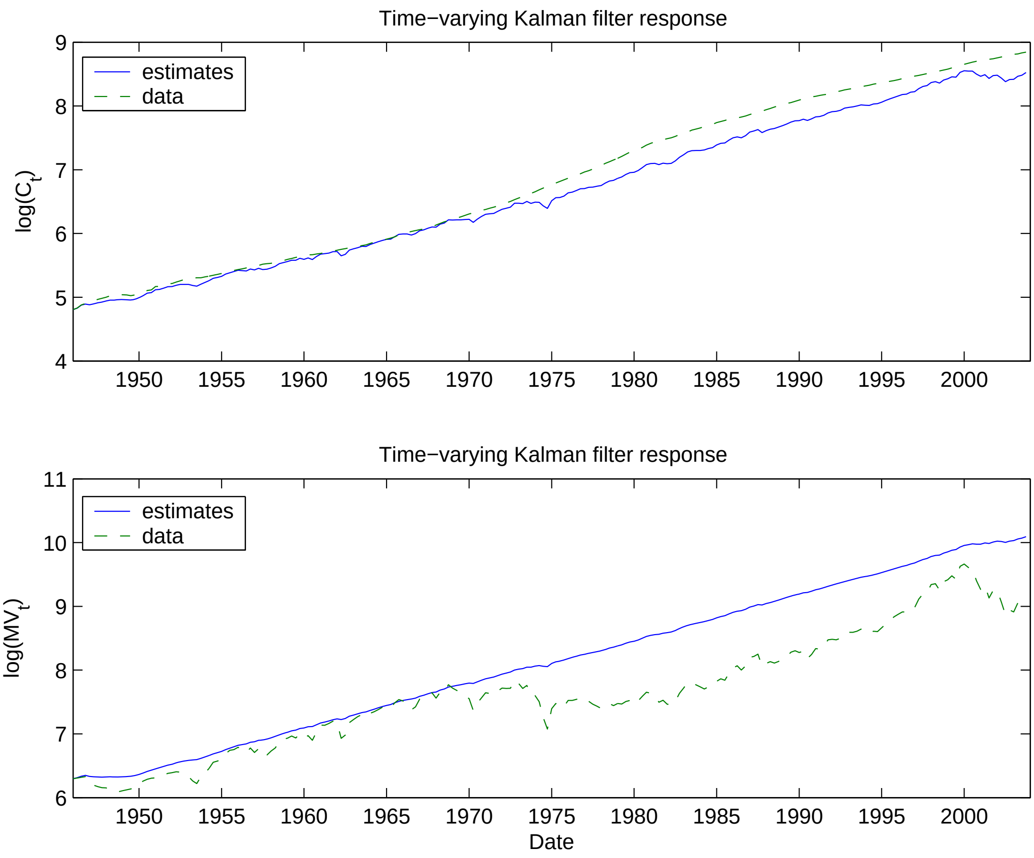

Figure 6: Kalman Filter Response of Consumption Growth and Growth of Market Value in the Model of Hall (2001)

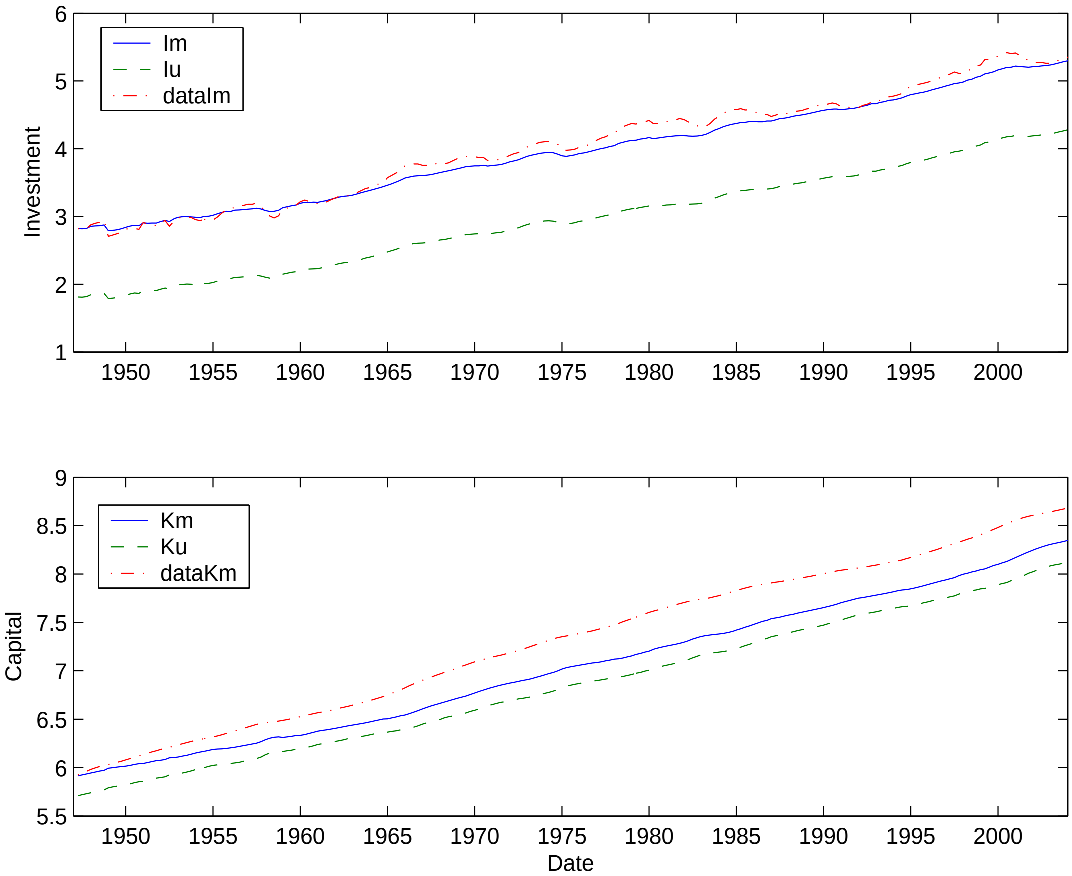

Figure 7: Estimates of Investment and Capital in the Two-Sector Model (in logs, using macroeconomic aggregate variables)

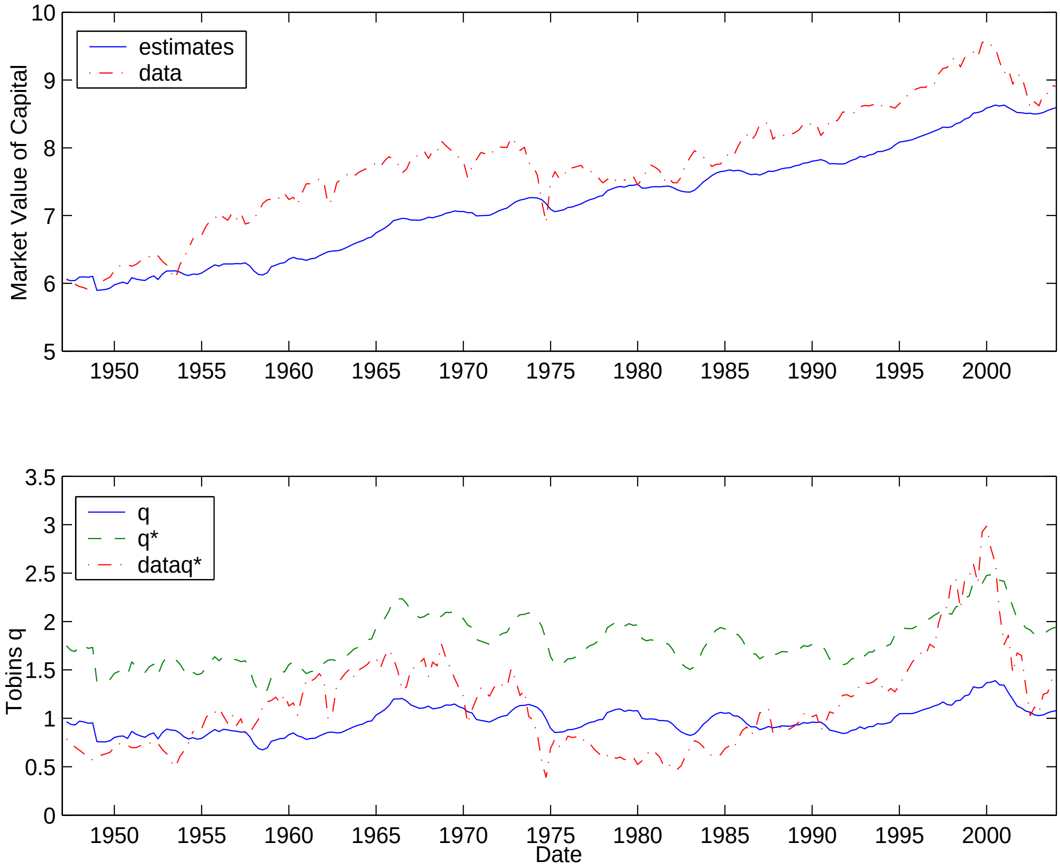

Figure 8: Estimates of Market Value of Capital (in logs) and Tobin's q in the Two-Sector Model (using macroeconomic aggregate variables)

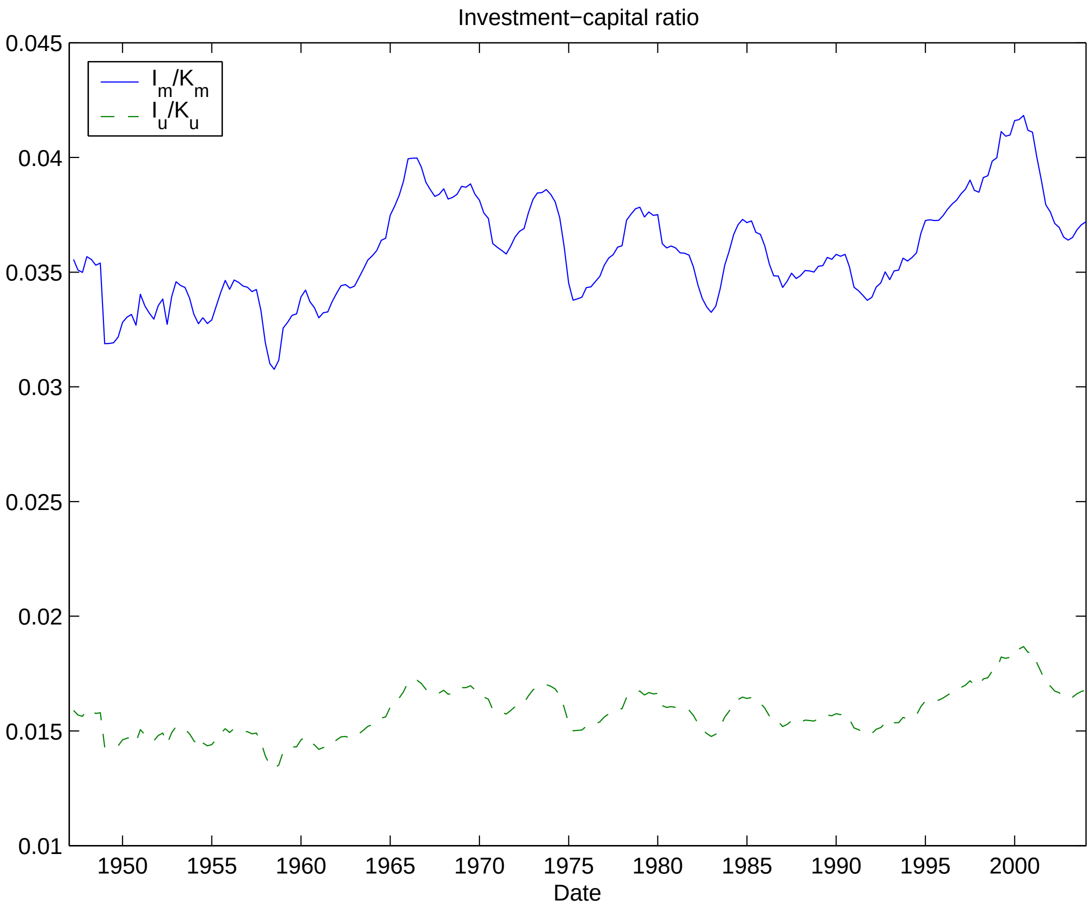

Figure 9: Investment-Capital Ratio in the Two-Sector Model (using macroeconomic aggregate variables)

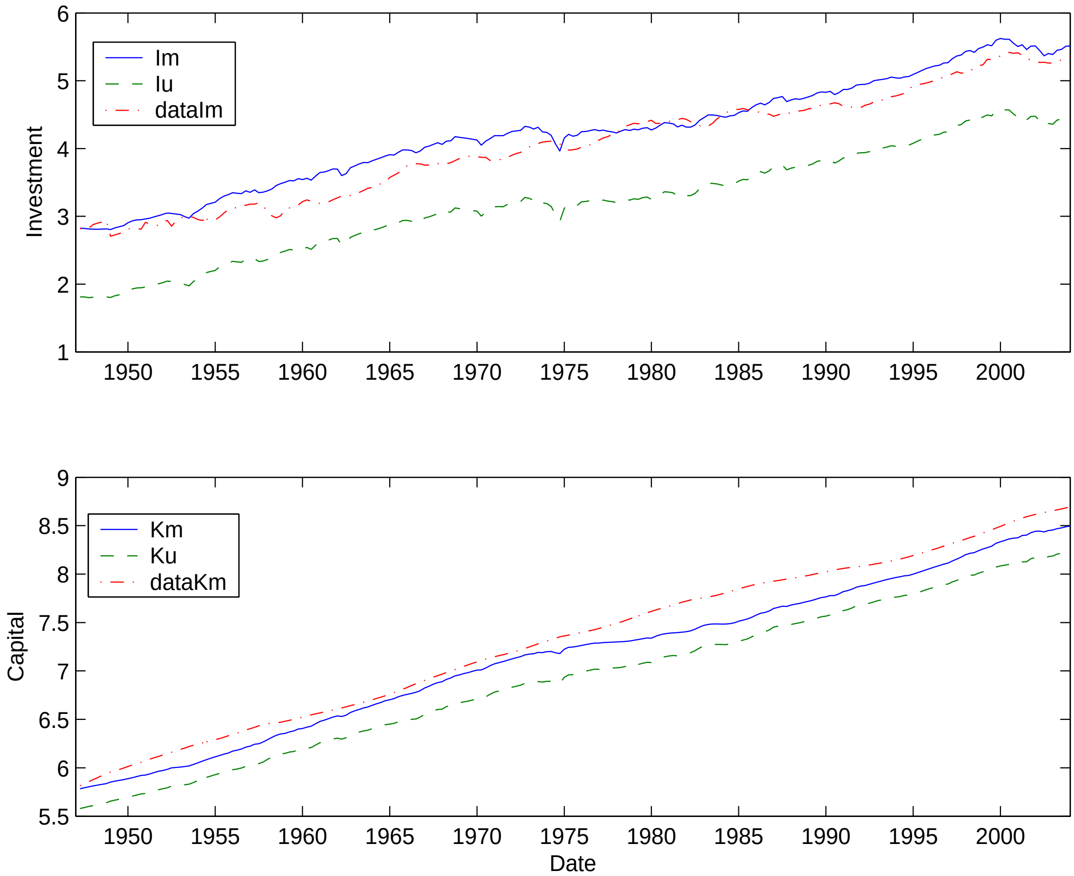

Figure 10: Estimates of Investment and Capital in the Two-Sector Model (in logs, using both macroeconomic aggregate variables and market value of capital)

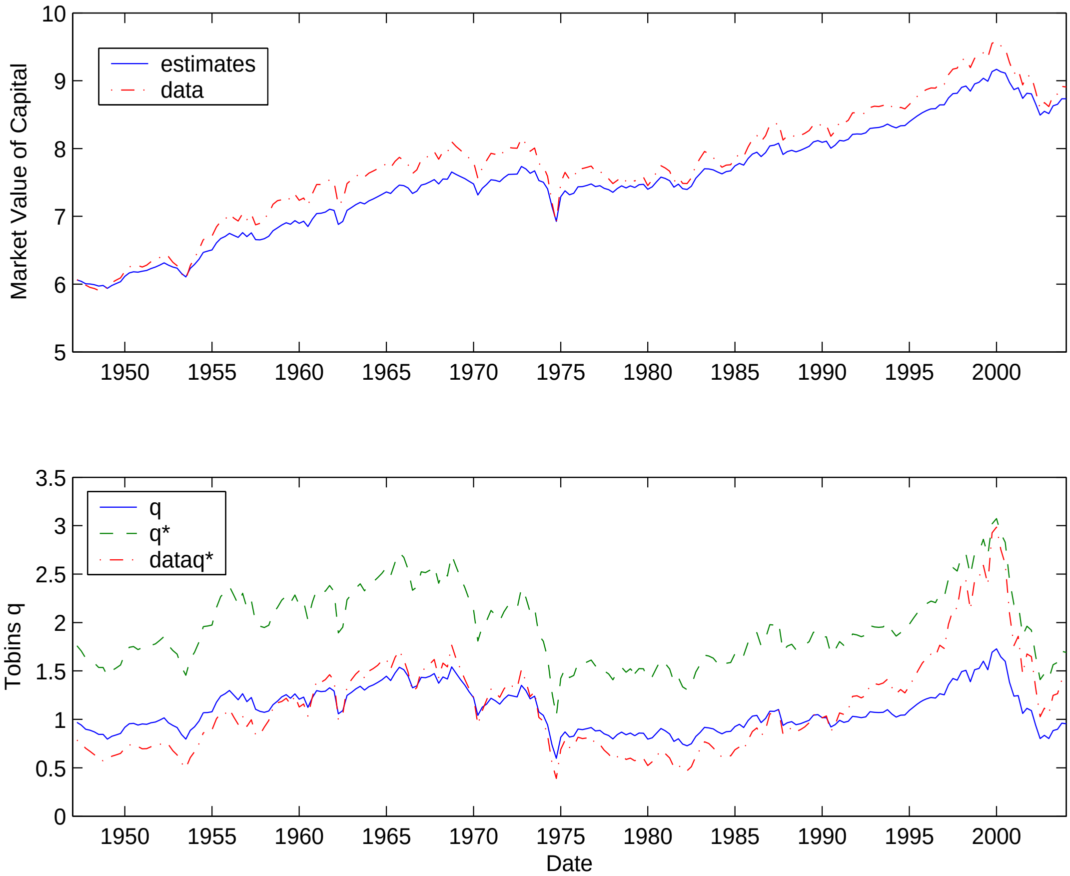

Figure 11: Estimates of Market Value of Capital (in logs) and Tobin's q in the Two-Sector Model (using both macroeconomic aggregate variables and stock prices)

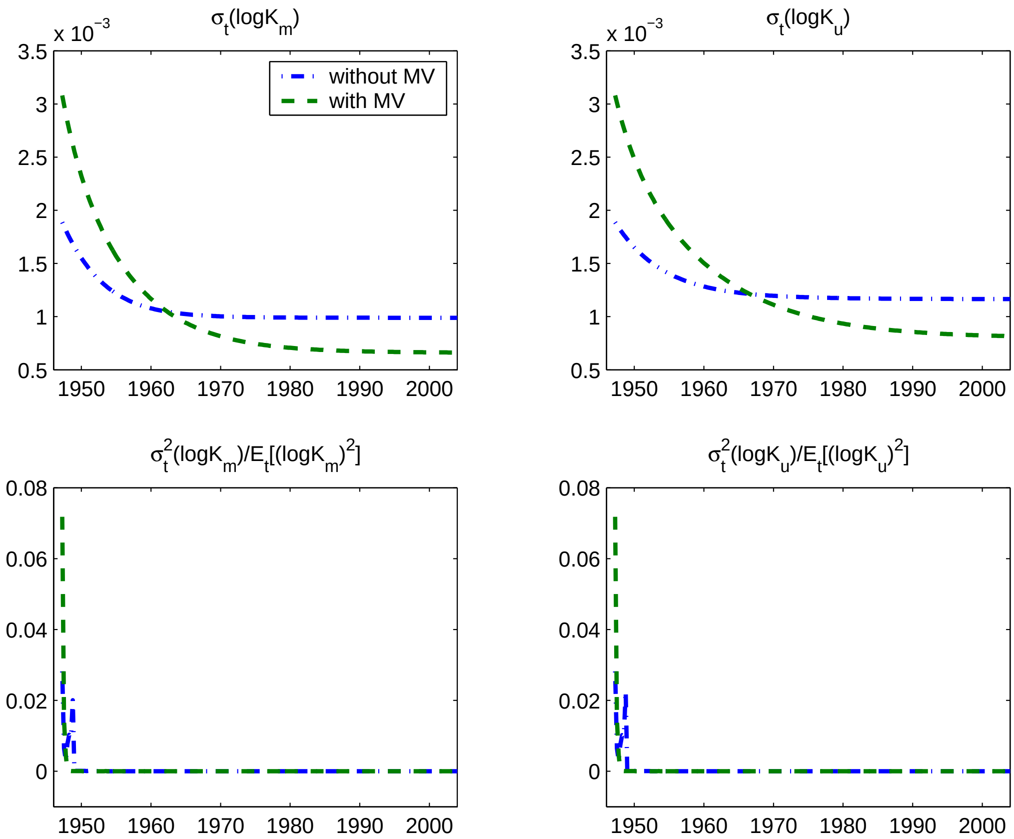

Figure 12: Accuracy of Measurement for Two-Sector Model

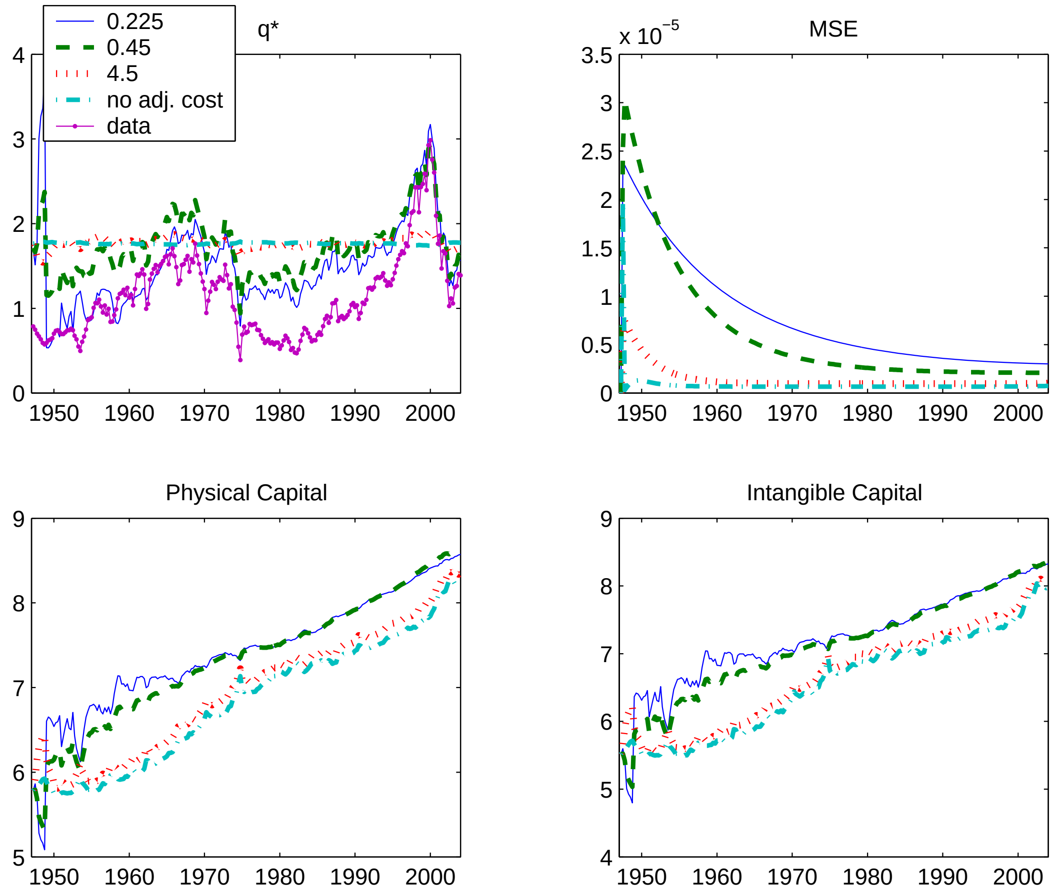

Figure 13: q^*, MSE and Capital Stock for Different Values of ξ.